KENDRIYA VIDYALAYA SANGATHAN

KOLKATA REGION

RESOURCE MATERIAL FOR

CLASS XII–MATHEMATICS

WORKSHOP

ON

‘Developing resource material for teaching of Mathematics for classes XI-XII based on new syllabus (2019-20)’

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Venue : **KENDRIYA VIDYALAYA, KV SALT LAKE-1**

OUR PATRONS

MS.P.B.S USHA , DC , KOLKATA REGION

COURSE DIRECTOR

MR.S.KARAN, PRINCIPAL,KV NO-1 SALTLAKE

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MR.BISWAJIT PATRA,PGT,KVNO-2 KHARAGPUR

SPECIAL ASSISTANCE

MRS.PRATIMA MANDAL ,PGT,KV SALTLAKE-1

MR.SUJAY BHAKAT,COMPUTER INSTRUCTOR, KV SALTLAKE-1

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41	SUMANA SAHA	GANDHI NAGAR
42	MR.BINAY RANJAN JANA	COSSIPORE
43	MR.RITU CHATTERJEE	ASANSOL
44	MR.RITU CHATTERJEE	ASANSOL

CHAPTER 1

Relations and Functions

LEARNING OBJECTIVES/OUTCOMES

1. RELATION ,TYPES OF RELATIONS
2. EQUIVALENCE RELATION
   A Relation which is Reflexive, Symmetric and Transitive.
3. FUNCTION
   To recall - identity , constant, linear, polynomial, rational, Signum, modulus, greatest integer functions.
4. TYPES OF FUNCTIONS
5. COMPOSITION OF FUNCTIONS
6. INVERSE OF A FUNCTION
   Problems based on linear, quadratic and rational functions.

KEY ITEMS

1. A relation R in set A is called an equivalence relation in set A if it is
   (a) reflexive i.e (x,x)ϵR , $\forall x \epsilon A$
   (b) symmetric i.e if (x,y) ϵR =>(y,x)ϵR , $\forall x, y \epsilon A$
   (c)transitive i.e if (x,y) ϵR &(y, z) ϵR =>(x,z)ϵR , $\forall x, y, z \epsilon A$
2. If n(A)= m, then number of relation in A is 2^{n^2}.
   Number of reflexive relation 2^{n^2-n}.
3. A function f:A→ B is invertible(bijective) if it is
   (a) one-one (injective) i.e f(x_1) = f(x_2)=>x_1 = x_2, x_1,$x_2 \epsilon A$,
   (b) onto(surjective) i.e f(x) = y defined for all y ϵ *codomain*
   Or, fog(x)= gof(x) = x
4. f(x)=y and f is invertible then x=f^{-1}(y) .

To be a successful we need to have 3 D's that are dedication, determination and devotion to reach a destiny.

THREE LEVELS OF GRADED QUESTIONS

Level I.

1. Check whether the relation R defined in the set { 1, 2, 3, 4} as R = {(1, 2), (2, 2), (1, 1), (4, 4),(1, 3), (3, 3), (3, 2)} is Reflexive, Symmetric and Transitive.
2. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither Reflexive nor Transitive.
3. Show that R defined as R = {(a, b): a ≤ b, a, b ∈**R**} is Reflexive and Transitive but not Symmetric.
4. Check whether the following functions are one-one and onto.
   a) f: **N→N** given by $f(x) = x^2$
   b) f: **R→R** given by $f(x) = x^2$
5. If the functions f: **R→R** given by $f(x) = x^2 + 3x + 1$ and g : **R→R** is given by $g(x) = 2x - 3$, find $f \circ g$ and $g \circ f$
6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B.Check whether f is one-one or not.
7. If f(x) = x + 7 and g(x) = x – 7, x∈**R**, find $f \circ g(7)$
8. If f:**R→R** is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$
9. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): |a – b| is a multiple of 4 } is an equivalence relation.
10. Check whether the function f: **R→R** defined by f(x) = 4+ 3x is one – one and onto. If so, find the inverse of f.
11. Let $f: R \to R$ be defined as f(x) = 10x +7. Find the function $g : R \to R$ such that g o f = f o g = I_R.

Level II

1. Check whether the relation R defined in the set { 1, 2, 3, 4, 5 , 6} as R = {(a, b): b = a+ 1} is Reflexive, Symmetric and Transitive.
2. If f : **R→R** be given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$, then find $f \circ f(x)$
3. Let f :**R** –{-3/5)→**R** defined by $f(x) = \frac{2x}{5x+3}$ is invertible. Find inverse of f.

4. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2} is an equivalence relation.
5. Let A and B be sets. Show that f : A x B→B x A such that f(a, b) =(b, a) is bijective function.
6. Let f : N →N be defined by f(x)=$\begin{cases}\frac{n+1}{2} if n is odd\\ \frac{n}{2} if n is even\end{cases}$ for every n ∈ N. Check whether the function is bijective.
7. Consider f:R_+→ $[4,\infty)$ given by f(x) = $x^2 + 4$. *Showthat* f is invertible with the inverse f^{-1} of f given by f^{-1}(y) = $\sqrt{y-4}$ where R_+ is the set of all non- negative real numbers.

Level III

1. If f: R →R is defined by f(x) = $x^2 - 3x + 2$, find f(f(x))
2. Find the number of binary operations on the set {a, b, c}.
3. If R_1 and R_2 are Equivalence Relation in a set A, Show that $R_1 \cap R_2$ is also an equivalence relation.
4. Show that the function f: R → {x∈R :-1 < x < 1} defined by f(x) = $\frac{x}{1+|x|}$, x ∈R is one- one and onto function.
5. Let f: N → R be a function defined as f(x)= $4x^2 + 12x + 15$. Show that f: N→Range f is invertible. Also find the inverse of f.

ONE MARK QUESTIONS WITH ANSWERS

01 mark questions

1. A set A has 3 elements and B has 4 elements, then number of injective functions from A to B is

   (a) 144 , (b) 12, (c) 24, (d) 64
2. Set A has 3 elements ,the number of relations defined in A is

   (a)27, (b)281, (c)512, (d)24
3. Domain of the function in real number defined by $f(x)=\sqrt{4-x^2}$ is

   (a) (-2,2) (b) [-2,2) (c) [-2,2] (d) (-2,2]
4. R= {(a,b),(b,a),(a,a)} be relation in set A = {a,b,c}, then R is

   (a) Reflexive (b) symmetric (c) transitive (d) equivalence.
5. Write a relation which is reflexive ,transitive, but not symmetric.
6. Check wheather $f(x)=x^2$ surjective or not in the set of natural number.

7. If $f(x)= \frac{1}{1-x}$, x≠ 1 then find fof.
8. State the reason for the relation R in the set {1,2,3} given by R= {(1,2), (2,1)} is not a transitive.
9. The relation R = {(a,b): $|a-b|$ is multiple of 3} in the set A = {1,2,3,…,10} is any equivalence relation. Write the equivalence classes related to 2.
10. f:{1,2,3}→ {1,2,5} and g:{1,2,5}→ {1,3} be given by f={(1,2),(3,5),(4,1)} and g={(1,3),(2,3),(5,1)} , write down gof.

ANSWERS

1.c, 2. c, 3. c, 4.c 5. In set A ={1,2,3} , R={(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)}, 6. No, since 2 has no pre image 7. $\frac{x-1}{x}, x \neq 0$, 8. (1,1) not in R, 9. {2,5,8} 10.{(1,3), (3,1),(4,3)},

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CHAPTER 2

Inverse Trigonometric Functions

KEY POINTS

What is an Inverse Function?

If y=f(x) and x=g(y) are two functions such that f (g(y)) = y and g (f(y)) = x, then f and y are said to be inverse of each other i.e.,

$g = f^{-1}$

If y = f(x) then $x = f^{-1}(y)$

The inverse trigonometric functions are the inverse functions of the trigonometric functions written as $\cos^{-1} x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$, $\text{cosec}^{-1} x$, $\sec^{-1} x$.

if y = sin x , then $x = \sin^{-1} y$, similarly for other trigonometric functions. This is one of the inverse trigonometric formulas. Now, $y = \sin^{-1}(x)$, $y \in [\pi/2 , \pi/2]$ and $x \in [-1,1]$.

Domain & Range of Inverse Trigonometric Functions

functions	Domain	Range
$\sin^{-1}x$	[-1,1]	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$\cos^{-1}x$	[-1,1]	$[0,\pi]$
$\tan^{-1}x$	R	$(-\frac{\pi}{2},\frac{\pi}{2})$
$\cot^{-1}x$	R	$(0,\pi)$
$\text{Sec}^{-1}x$	R – (-1,1)	$(0,\pi]-\{\frac{\pi}{2}\}$
$\text{Cosec}^{-1}x$	R – (-1,1)	$[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$

$\sin^{-1}(\sin\theta)=\theta$, $\cos^{-1}(\cos\theta)=\theta$ within the domain

$\sin^{-1}(-x)=-\sin^{-1}(x)$, $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$, $\tan^{-1}(-x)=-\tan^{-1}x$.

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$, $\text{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2}$

THREE LEVELS OF GRADED QUESTIONS

Level I

1. Find the principal value of $\sin^{-1}(\frac{-1}{2})$
2. Find the principal value of $\cos^{-1}(\frac{\sqrt{3}}{2})$
3. Find the principal value of $\tan^{-1}(-\sqrt{3})$
4. Find the value of $\tan^{-1}(1)+\sin^{-1}(\frac{-1}{2})$
5. Find the value of $\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$
6. Find the value of $\sin(\frac{\pi}{3}-\sin^{-1}(\frac{-1}{2}))$
7. Find the principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$
8. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$
9. Solve : $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$
10. Solve :$2\tan^{-1}\frac{1-x}{1+x}=\tan^{-1}x, x>0$

Level II

1. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}(\frac{-1}{2})$
2. Find the value of $\tan(\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}(\frac{3}{2}))$
3. Write in simplest form: $\tan^{-1}(\frac{\cos x - sinx}{cosx + sinx})$
4. Find the value of $\tan\frac{1}{2}(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}), |x| < 1, y > 0\ and xy < 1$
5. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$
6. Prove that $\cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-\sin x}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right) = \frac{x}{2}$
7. Prove that $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}(\frac{3x-x^3}{1-3x^2})$

Level III

1. If $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, find the value of x.
2. Find the value of $\cos^{-1}\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) + \text{cosec}^{-1}(\frac{1+\sqrt{x}}{1-\sqrt{x}})$
3. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{60} = \pi$
4. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x\ ,\ \frac{-1}{\sqrt{2}} \le x \le 1$
5. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then show that $xy + yz + zx = 1$
6. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$
7. Solve for x: $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$
8. Prove that$\cos(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}) = \frac{6}{5\sqrt{13}}$
9. Solve:$\cos(\tan^{-1} x) = \sin(\cot^{-1}\frac{3}{4})$
10. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$
11. Simplify: $\sin^{-1}(\frac{3x+4\sqrt{1-x^2}}{5})$
12. Prove that $\sin[\cot^{-1}(\cos\tan^{-1} x)] = \sqrt{\frac{x^2+1}{x^2+2}}$
13. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$ prove that a + b + c = abc
14. Show that : $2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right) = \tan^{-1}\frac{\sin\alpha.\cos\beta}{\cos\alpha + \sin\beta}$

ONE MARK QUESTIONS WITH ANSWERS

1. Find the value of $\sin^{-1}(\sin\frac{3\pi}{5})$ Ans . $\frac{2\pi}{5}$

2. Evaluate $\cos^{-1}(\cos\frac{7\pi}{6})$ Ans. $\frac{5\pi}{6}$

3. Which is greater $\tan 1$ or $\tan^{-1} 1$. Ans. $\tan 1 > \tan^{-1} 1$

4. $\sin^{-1}(\sin\frac{2\pi}{3}) = \frac{2\pi}{3}$, state true or false. False

5. Domain of function $\sin^{-1} x$ is (-1,1), state true or false. False

6. If $\tan^{-1} x = \sin^{-1}\frac{1}{\sqrt{2}}$, then x is ___ . Ans. 1

7. Find the value of $\cot(\tan^{-1} a + \cot^{-1} a)$ Ans. 0

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CHAPTER3

MATRICES

Key Points

* A matrix is an ordered rectangular array of numbers or functions.

* A matrix having m rows and n columns is called a matrix of order $m \times n$.

* An $m \times n$ matrix is a square matrix if $m = n$.

* $A = [a_{ij}]_{m\times m}$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$.

* $A = [a_{ij}]_{n\times n}$ is an identity matrix, if $a_{ij} = 1$, when $i = j, a_{ij} = 0$, when $i \neq j$.

* $kA = k[a_{ij}]_{m\times n} = [k(a_{ij})]_{m\times n}$

* $(k + l)A = kA + lA$, where $k$and l are constant.

* If $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$, then $AB = C = [C_{ik}]_{m\times p}$, where $C_{ik} = \sum_{j=1}^{n} a_{ij}$.

* Transpose of a matrix. A matrix obtained by interchanging rows and columns of matrix A is called transpose of matrix i,e. A^{l}

* A is a symmetric matrix if $A^I = A$

* A is a skew symmetric matrix if $A^I = -A$

* Any square matrix can be represented as the sum of a symmetric and skew symmetric matrix.

$$A = \frac{1}{2}[A + A^I] + \frac{1}{2}[A - A^I]$$

Elementary operations of a matrix as follows

(1) $R_i \leftrightarrow R_j \ \ or \ \ C_i \leftrightarrow C_j$

(2) $R_i \rightarrow kR_j \ \ or \ \ C_i \rightarrow kC_j$

(3) $R_i \rightarrow R_i + kR_j \ \ \ or \ \ \ C_i \rightarrow C_i + kC_j$

THREE LEVELS OF GRADED QUESTIONS

S.No. LEVEL- 1

1. If a matrix has 5 elements, write all possible order it can have?
2. If $\begin{bmatrix}3 & 4\\ 2 & x\end{bmatrix}\begin{bmatrix}x\\ 1\end{bmatrix} = \begin{bmatrix}19\\ 15\end{bmatrix}$, find the value of x.
3. Construct a 3x3 matrix A, where a_{ij} = 2i – 3j .
4. If order of matrix A is 2x3 and order of matrix B is 3x4 , find the order of AB.
5. Write the value of $x + y + z$ if $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ -1\\ 0\end{bmatrix}$
6. For what value of x, is the matrix A=$\begin{bmatrix}0 & 1 & -2\\ -1 & 0 & 3\\ x & -3 & 0\end{bmatrix}$ a skew-symmetric matrix?
7. If A=$\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 5\end{bmatrix}$ and B= $\begin{bmatrix}3 & -1\\ 2 & 2\\ 1 & 3\end{bmatrix}$, write the order of AB and BA.
8. If A, B and C are three non-zero square matrices of same order, find the condition on A such that AB = AC ⇒ B = C.

9	Give an examples of two non-zero 2×2 matrices A and B such that AB=0.
10	Find the value of x and y if $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$
11	If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} and B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}, then find A^T - B^T.$
12	If $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$, then find x and y.
13	If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find A + A ′
14	If $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}'$, then find x,y and z.
15	If A is a matrix of order 2×3 and B is of matrix of order 3×5, what is the order of the matrix (AB)′ ?
16	Using elementary operation , find the inverse of the matrix A $=\begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$
17	If $\begin{pmatrix} 2x & 1 \\ 5 & x+2y \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix}$, find the value of y .
18	If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} and B = \begin{pmatrix} -3 & 2 \\ 0 & 1 \end{pmatrix}$,find i)3A+2B ii)A- 3B

Sl.No	**LEVEL II**
1	If A is square matrix satisfying $A^2 = I$, then what is the inverse of A?
2	If A $=\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix} issymmetric Find x.$
3	Find the product $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4]$
4	If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric, find a + b + c .
5	If A $=\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$ and B $= \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$, 2A + B + X = 0 , find X.

6 If 2 A + 3 B = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3 A + 2 B = $\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$, find A and B.

7 If A = $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$, then verify that (AB)′= B′A. ′

8 Find A such that A $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ = $\begin{bmatrix} 0 & -3 \\ 10 & 3 \end{bmatrix}$.

9 Show that A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies $A^2 - 4A - 5I = 0$, and hence find A^{-1}.

10 If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, Prove that A + A ′ is a symmetric matrix.

11 Using elementary operation , find the inverse of the matrix A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

12 Find the matrix X, for which $\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ X = $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

13 Let A = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as a sum of symmetric and skew symmetric matrices.

14 If A ′= $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find A ′- B ′

15 If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that f(A) = 0.

16 Solve for x and y: $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$.

17 If A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 = 8A + k$

18 Express the following matrix as the sum of a symmetric and skew symmetric matrix and Verify your result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

19 Using elementary operations, find the inverse of the following matrix, : $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

20 If A = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix}$, verify that (AB) ′= B ′A ′

21 If A = $\begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$ prove that A^n = $\begin{bmatrix} cosnx & sinnx \\ -sinnx & cosnx \end{bmatrix}$ for all positive integers n

LEVEL III

1 Find the inverse of the following matrix using elementary operations:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

2 If A^{-1} = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$

3 Find X, if X + $\begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix}$ = $\begin{bmatrix} 2 & 4 \\ 5 & 0 \end{bmatrix}$.

4 Find x, if $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix}$ = 0.

5 If A = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$,Prove that A+ A ′is symmetric and A – A ′ is skew symmetric matrices

6 Using elementary operation, find the inverse of the matrix

A = $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

7 If A = $\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ -1 & 4 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute A^2B^2

8 If f(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that f(x) f(y) = f (x +y).

9 If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, find a,b and k.

10 If AA′= I where A = $\begin{bmatrix} b & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ find a,b and c.

11 If A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and A^2 = $\begin{bmatrix} \propto & \beta \\ \beta & \alpha \end{bmatrix}$ Then prove that α + β = (a + b)^2.

12 If A = $\begin{bmatrix} cos\theta & isin\theta \\ isin\theta & cos\theta \end{bmatrix}$, then prove by principle of mathematical induction that A^n = $\begin{bmatrix} cosn\theta & isinn\theta \\ isinn\theta & cosn\theta \end{bmatrix}$.

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CHAPTER 4

DETERMINANTS

KEY POINTS

* Determinant of a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

* A^I = Transpose of A

* If we interchange any two rows (columns) then sign of determinant changes.

* If any two rows (or Columns) are identical or proportional, then value of determinant is zero.

* If we multiply each element of a row or a column of a determinant by constant K, then value of determinant is multiplied by K.

* Multiplying a determinant by K means multiply elements of only one row (or columns) by K.

* If $A = [a_{ij}]_{3X3}$ then $|KA| = K^3|A|$

* If elements of a row or a column in a determinant can be expressed as sum of two or more elements, then the given determinant can be expressed as sum of two or more determinants.

* If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then value of determinant remains same.

* Area of a triangle with vertices $(x, y), (x_2, y_2)$ and (x_3, y_3) is given by $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

* Cofactor of a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$ where M_{ij} is the minor of an element.

* $A(adj\ A) = (adj\ A)A = |A|I$ where A is square matrix of order n.

* A square matrix A is said to be singular or non-singular according as $|A| = 0$ or $|A| \neq 0$

* $AB = BA = I$ where B is a square matrix, then B is called inverse of A.

* A square matrix A has inverse if and only if A is non singular.

Solving equations by matrix method

If $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$,$a_3x + b_3y + c_3z = d_3$;

Then these equations can be written as $AX = B$, where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$AX = B$ or, $X = A^{-1}B$ 0r, $X = \frac{Adj\ A}{|A|} \times B$ Provided $|A| \neq 0$

THREE LEVELS OF GRADED QUESTIONS

LEVEL I

1. $If\ A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then show that $|2A| = 4|A|$.

2 Find the value of x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.

3 Using property prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$.

Using property prove that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$.

5 Find the area of the triangle with vertices (-2, -3), (3,2) and (-1, -8) .

6 Using co-factors of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

7 Using co-factors of elements of third column evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

8 Find the adjointof $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

9 Find the inverse of $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$.

10 If A is of order 3x3 and $|A| = 5$ then find the $|adjA|$.

11 If $A_{ij}$is the co factor of the element a_{ij} if the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$write the value of $a_{32} \cdot A_{32}$

LEVEL –II

1 If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27\ |A|$.

2 Using properties evaluate $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$.

3 Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ = 4 abc.

4 Using properties show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ = (a-b)(b-c)(c-a).

5 Solve 5x+2y =4 ; 7x+3y = 5.

6 Find the equation of the line joining the points (1,2) and (3,6) using determinants.

7 Verify A(adjA) = (adj A) A $= |A|$ I if A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$.

8 If A = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1} A^{-1}$.

9 If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.

13 Show that the points A (a, b+c), B (b,c+a), C(c,a+b) are collinear.

14 Find values of k if area of triangle is 4 sq.units and vertices are (-2,0),(0,4), (0,k).

15 Find equation of line joining (1,2) and (3,6) using determinants.

16 Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$ $= a^3$

17

Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ $= abc + bc + ca + ab = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$

18. Using properties of determinants prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

19 Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

20 Using properties of determinants, solve the following for x :

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

21 Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

22 If x, y, z are the 10th, 13th and 15th terms of a G.P. find the value of

$$\begin{vmatrix} \log x & 10 & 1 \\ \log y & 13 & 1 \\ \log z & 15 & 1 \end{vmatrix}$$

LEVEL III

1 If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations:

$x + y + 3z = 3$, $4x - y = 3$ and $-7x + 2y + z = 2$.

2 If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} and B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB. Use this to solve the following system of equations. x - y =3 , 2x +3y + 4z =17 and y + 2z = 7 .

3 Examine the consistency of the following system of equations.

3x-y+7y=3, 2x+y+3z, x+4y-2z

4 Examine the consistency of the followi8ng system of equations

x –y + z=3, 2x+y-z=3, -x-2y+2z=1

5 If A is a matrix of order 3 X 3 then verify that $\left(A^2\right)^{-1} = \left(A^{-1}\right)^2$

6 If A is a square matrix of order 3 such that $|adjA| = 64,\ find|A|$

7 If A and B are non singular square matrices of the same order, then write the relationship between adj AB, adj A and adj B.

8 If A is invertible matrix of 3 X 3 and $|A| = 7$ then find $|A^{-1}|$

9 If a, b and c are real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$. Show that either a+b+c=0 or a=b=c.

10 Using properties of determinants, prove that if x, y and z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that 1+xyz=0.

11 Using properties of determinants, prove the following:

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2\text{abc}(a+b+c)^3.$$

12 Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

13 Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to Solve the system of equations:

x – y +2z = 1,2y – 3z = 1, 3x – 2y + 4z = 2.

14 Using matrices solve the following system of equations:

2x – 3y +5z = 11, 3x + 2y -4z = -5, x + y -2z = -3

15 Use matrix method, solve the following system of equations:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$: where $x \neq 0, y \neq 0, z \neq 0$

16 Two institutions decided to award their employees for the three values of Resourcefulness, competence and determination in the form of prizes at the rate Rs x, Rs y and Rs z respectively per person. The first institution decided to award 4, 3 and 2 respectively with total prize money of Rs37000 and the second institution decided to award respectively 5, 3 and 4 with total prize money of Rs47000. If all the three prizes per person together amount Rs 12000, then using matrix method find the values of x, y and z.

1 MARK QUESTIONS

1. A matrix A is both symmetric and skew symmetric if A is

   (a) Null matrix (b) unit matrix (c) row matrix (d) none of these

2. If A and B are invertible matrix then $(AB)^{-1}$ = ...

   (a) $A^{-1}B^{-1}$ (b) AB (c) $B^{-1}A^{-1}$ (d) none of these

3. If A be any invertible matrix such that $A^3 = I$ then A^{-1} = ...

   (a) I (b) A^2 (c) A^3 (d) A^4

4. If A is a matrix of order 3x3 , then $(A^2)^{-1} = (A^{-1})^2$ is

   (a) True (b) false (c)none of these

5. If matrix $A = \begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, then |A. adjA| = ...

   (a) 9 (b) 3 (c)27 (d) 12

6. If A is any invertible matrix then A is

   (a) square matrix (b) non singular matrix (c) both (d) none of these

7. If A is any skew symmetric matrix then A' =

   (a) A (b) - A (c) A^2 (d) none of these

8. Let A be a square matrix of order 3x3 then| kA| = ...

   (a) k |A| (b) $k^2|A|$ (c) $k^3|A|$ (d) 3k|A|

9. which of the following is correct

   (a) determinants is a square matrix

   (b) determinants is a number associated to a matrix

   (c) determinants is a number associated to a square matrix

(d) none of these

10. if A is any invertible matrix of order 2, then det(A^{-1}) is equal to

(a) det(A) (b) $\frac{1}{\det(A)}$ (c) 1 (d) 0

11. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, find the values of x and y.

If $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$, find the value of x.

13. Write the adjoint of the following matrix : $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

14. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, Write A^{-1} in terms of A.

15. A matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then write the value of p.

16. If a matrix has 5 elements, write all possible orders it can have.

17. Write the order of the product matrix: $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$.

18. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

19. Show that all the elements on the main diagonal of a skew symmetric matrix are zero.

20. Find the value of $x + y$ from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

21. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

22. If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the value of a.

23. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

24. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$.

25. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.

26. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the value of a and b.

27. If $\begin{bmatrix} 15 & x+y \\ 2 & y \end{bmatrix} = \begin{bmatrix} 15 & 8 \\ x-y & 3 \end{bmatrix}$. Find the value of x.

28. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, the find the value of k.

29. Find the value of x and y: $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

30. Solve for x and y : given that $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

31. Find the cofactor of a_{12} in the following: $\begin{bmatrix} 1 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

32. Find the minor of a_{23} in the following: $\begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

33. Find the value of k, such that $\begin{vmatrix} k & 5 \\ 4 & 2k \end{vmatrix} = \begin{vmatrix} 5 & -2 \\ 16 & 4 \end{vmatrix}$

34. Find the value of x from the following: $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$.

35. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|\text{adj A}|$.

36. If A is square matrix of order 3 such that $|\text{adj A}| = 64$, find $|A|$.

37. If A is a square matrix of order 3 and $|3A| = k|A|$, then find the value of k.

38. A matrix A of order 3×3 has determinant 22. What is the value of $|4A|$.

39. If $A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$, then find the determinant of the matrix $A^2 - A$.

40. Using determinants, find the equation of the line passing through the points $(1, 2)$and $(3, 6)$.

41. Find the value of λ so that the points $(1, -5), (-4, 5)$and $(\lambda, 7)$ are collinear.

42. Evaluate the determinant $\begin{vmatrix} \sin 75^0 & \cos 15^0 \\ -\cos 75^0 & \sin 15^0 \end{vmatrix}$

43. If $A = \begin{bmatrix} 5 & 3 \\ 10 & -2 \end{bmatrix}$, then show that $|5A| = 25|A|$.

44. For what value of k, the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible ?

45. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular ?

46. If $A = \begin{bmatrix} x+2 & 3 \\ x+5 & 4 \end{bmatrix} = 3$, then find the value of x.

47. If A is a matrix of order m×3 and B is a matrix of order 3× p then what will be the order of matrix AB ?

48. Find the number of all possible matrices of order 3×3 with each entry 0 or 1.

49. If A is a square matrix of order 3 such that$|\text{adj}A| = 9$, find$|A^T|$.

50. Find det (A) , if A = $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

ANSWERS

1. a 2. C 3.b 4.a 5.b 6.c

7. b 8.c 9.c 10.b 11. x = 7 , y = -1

12. X = 1 13. $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ 14. $\frac{1}{19}$A 15. $p = 4$

16.1x5, 5x1 17. 1x1 18.Prove it 19. Prove it 20.X= 2, y= 9

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Chapter 5
Continuity and Differentiability

Key Points

1. A function is said to be continuous at x=a if LHL = RHL = f(a)
2. A function is said to be continuous in interval I if it is continuous at every point of the interval.
3. A function is discontinuous at x= a if,

(i) f(a) does not exist

(ii) limit does not exist

(iii) both exist but are not equal

4. Polynomial function, absolute value function, exponential function, logarithmic function, sine, cosine, tangent, rational function defined within their domains are continuous.
5. Let f and g are two continuous functions then

   (i) f + g

   (ii) f – g

   (iii) fg (iv) f/g, g(x)≠0 are also continuous
6. Let f and g are real functions such that fog is defined at x = a. if g is continuous at x = a and f is continuous at g(a) then fog is continuous at x = a.
7. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, h is very small positive number.

IMPORTANT FORMULAS:

1. $\frac{d\,(x^n)}{dx} = n.\ x^{n-1}$ 2. $\frac{d}{dx}(1/x) = -1/x^2$

3. $\frac{d}{dx}(e^x) = e^x$ 4. $\frac{d}{dx}(a^x) = a^x \log a$

5. $\frac{d}{dx}\log x = 1/x$ 6. $\frac{d}{dx}(\sin x) = \cos x$

6. $\frac{d}{dx}(\cos x) = -\sin x$ 7. $\frac{d}{dx}(\tan x) = \sec^2 x$

8. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$ 9. $\frac{d}{dx}(\sec x) = \sec x.\ \tan x$

10. $\frac{d}{dx}(\mathrm{cosec}\ x) = -\mathrm{cosec}\ x.\ \cot x$ 11. $\frac{d}{dx}\sin^{-1}x = 1/\sqrt{1-x^2}$

12 $\frac{d}{dx}\cos^{-1}x = -\ 1/\sqrt{1-x^2}$ 13. $\frac{d}{dx}\tan^{-1}x = 1/(1+x^2)$

14. $\frac{d}{dx}\cot^{-1}x = -\ 1/(1+x^2)$ 15. $\frac{d}{dx}\sec^{-1}x = 1/\ (x\sqrt{x^2-1}\ )$

16. $\frac{d}{dx}\mathrm{cosec}^{-1}x = -\ 1/\ (x\sqrt{x^2-1}\ )$

THREE LEVELS OF GRADED QUESTIONS

Level I

1 Examine the continuity of f(x) = x - 5

2 Examine the continuity of f(x) = $\frac{1}{x-3}$, $x \neq 3$

3 Find the value of k so that f(x)= $\begin{cases} kx^2, if x \leq 2 \\ 3 \; if x > 2 \end{cases}$ is continuous at x=2

4 If y=(logx)^2 find $\frac{dy}{dx}$

5 If y= e$^{\sin x}$ find $\frac{dy}{dx}$

6 Find $\frac{dy}{dx}$ if 2x+3y=sinx

7 Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

8 Differentiate with respect to x : y=x^{logx}

9 Find $\frac{dy}{dx}$ if x=at^2 , y= 2at

10 If y= asinx+bcosx, then prove that y"+ y = 0

11 Find the derivative of (3x^2-9x+5)^9 with respect to x

12 Find the derivative of cos^{-1}(x/2) with respect to x

13 Find the derivativeof cos2x+cos5x-secx, with respect to x

14 Find the second order derivative of y=x^3 + tanx

15 Differentiate log(logx) with respect to x.

LEVEL II

1 If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, Show that $\frac{dy}{dx} = -\frac{y}{x}$

2 Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x

3 Differentiate sin(cos(x^2)) with respect to x

4 Differentiate $\cos x^3 \cdot \sin^2(x^5)$ with respect to x

5 Differentiate $\sec\left(\tan\left(\sqrt{x}\right)\right)$ with respect to x

6 If $y = \cos^{-1} x$, $find \frac{d^2y}{dx^2}$ in terms of y alone.

8 f $x^p y^q = (x+y)^{p+q}$, Prove that $\frac{dy}{dx} = \frac{y}{x}$

9 Find $\frac{dy}{dx}$ if y=$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, 0 <|$x$| < 1

10 Show that the function f(x)=|x + 2 | is continuous at every x∈ R, but fails to be differentiable at x=−2

11 If y=3cos(logx) + 4sin(logx), Show that x^{2}y$_2$ +xy$_1$ + y = 0

12 Use Lagrange's mean value theorem to determine a point P on the curve y=$\sqrt{x-2}$, where the tangent is parallel to the chord joining (2,0) and (3,1).

13 If $y = x^x + (\sin x)^x$, $find \frac{dy}{dx}$

LEVEL III

14 For $whatvaluesofaandb$, $thefunctionfdefinedas$:

$$f(x) = \begin{cases} 3ax + b, if x < 1 \\ 11, \quad if x = 1 \\ 5ax - 2b, if x > 1 \end{cases}$$ iscontinuous at x=1.

15 If $x^y + y^x = a^b$, $find \frac{dy}{dx}$

16 If $x^y + y^x + x^x = a^b$, $find \frac{dy}{dx}$

17 If $tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

18 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-x^2}{1-y^2}}$

Find the relationship between 'a' and 'b' so that the function 'f' defined by

$$f(x) = \begin{cases} ax + 1, & x \le 3 \\ bx + 3, & x > 3 \end{cases} iscontinuousatx = 3.$$

19 The function f(x) is defined as f(x) = $\begin{cases} x^2 + ax + b, 0 \le x < 2 \\ 3x + 2, \quad 2 \le x \le 4 \\ 2ax + 5b, \quad 4 < x \le 8 \end{cases}$.

if f(x) is continuous on [0,8], find the values of a and b.

20 If $x^y = e^{x-y}, provethat \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

21 If $y = x^{x^{x^{\cdots^{\infty}}}}$ find $\frac{dy}{dx}$

22 Find the value of k for which $f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} if -1 \le x < 0 \\ \frac{2x+1}{x-1} if\ 0 \le x < 1 \end{cases}$

is continuous at x = 0

23 Find the relationship between 'a' and 'b' so that the function 'f' defined by :

$$f(x) = \begin{cases} ax + 1, & x \le 3 \\ bx + 3, & x > 3 \end{cases} iscontinuousatx = 3.$$

24 Find all the point of discontinuity of the function f defined by

$$f(x)=\begin{cases} x+2, & if\, x \le 1 \\ x-2, & if\ 1 < x, 2 \\ 0\ , & if\, x \ge 2 \end{cases}$$

25 If $= \left(x+\sqrt{x^2+1}\right)^m$, show that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y=0$.

26 Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

27 IF $x=\sqrt{a^{\sin^{-1}t}}$ $and Y=\sqrt{a^{\cos^{-1}t}}$, $showthat\ \frac{dy}{dx}=-\frac{y}{x}$

28 Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2-1}}{x}\right]$ with respect to x.

ONE MARK QUESTIONS WITH ANSWERS

1) Which of the following function is continuous at x = 0

(i) $\frac{1}{x}$ (ii) $\sin\frac{1}{x}$ (iii) $\cos\frac{1}{x}$ (iv) x^2

2) If a function f(x) = x^k is continuous at x = k , then

(i) k < 0 (ii) k ≤ 0 (iii) k ≥ 0 (iv) k ≠ 0

3) The function f(x) = $\frac{x+2}{-1-x+2\,x^2}$ is discontinuous at the following

(i) -1 , $\frac{1}{2}$ (ii) -1 , $-\frac{1}{2}$ (iii) 1, $-\frac{1}{2}$ (iv) 1 , $\frac{1}{2}$

4) Find whether following is True or false?

If f and g be two any continuous function then $\frac{f}{g}$ is continuous .

5) Choose the correct options

(i) Sum of two continuous is not continuous.

(ii) Product of two continuous function is continuous.

(iii) If f is not defined at the point x= a, the f is continuous at x = a.

(iv) None of these

6) $f(x)=\begin{cases} 2x, & if\ x<2 \\ 2, & if\ x=2 \\ x^2 & if\ x>2 \end{cases}$

The function f(x) at x = 2

(i) Continuous

(ii) The limit of a function does not exist.

(iii) Limit exist but f(2) is not define .

(iv) None

7) $f(x) = \begin{cases} x-1, & if\ x < 2 \\ 2x-1 & if\ x \geq 2 \end{cases}$

At the point x = 2 , f is

(i)Continuous (ii) discontinuous (iii) f(2) is not define (iv) none

8) Which of the following function is differentiable at x =0

(i)$\sqrt{x}$ (ii)$\frac{1}{x}$ (iii) x^{-2} (iv) tanx

9) y= e^{sinx} then $\frac{dy}{dx}$ =

(i) –cosx e^{sinx} (ii) e^{sinx} sinx (iii) $e^{\sin\ x}$ (iv) e^{sinx} cosx

10) If $x^2+ y^2=1$ then $\frac{dy}{dx}$ is

(i) $\frac{x}{y}$ (ii)- $\frac{x}{y}$ (iii) $\frac{y}{x}$ (iv) - $\frac{y}{x}$

11) which of following is true

(i) continuity of a function implies the differentiability of a function

(ii) differentiability of a function does not imply the continuity of a function

(iii) differentiability of a function implies continuity of a function

(iv)None of this

12) y = x^x, then $\frac{dy}{dx}$ =

(i) x x^{x-1} (ii) x^x logx (iii)) x^x (1 + logx) (iv)) x^x(1 − logx)

13) The derivative of the function y = log(sinx) is

(i) tanx (ii) –tanx (iii) –tanx (iv) –cotx

14) y = sin(logx) then $\frac{dy}{dx}$ =

(i) –sin(logx) (ii) $\frac{1}{x}$ cos(logx) (iii))- $\frac{1}{x}$ sin(logx) iv)) $\frac{1}{x}$ sin(logx)

15) The derivative of a constant function is always

(i) zero (ii) non zero (iii) finite (iv) one

16) y = $\sin^3 x$, then $\frac{dy}{dx}$ =

i) $\sin^3 x \cos x$ ii) 3 $\sin^2 x \cos x$ iii) – 3 $\sin^2 x \cos x$ iv) 3sinxcosx

17) The derivative of $\frac{1}{x}$ at the point x=1

i)- 1 ii) 2 iii) does not exist iv) 0

18) Derivative of e^{logx} is equals to

i) e^{logx} ii) x $e^{log x}$ iii) $\frac{1}{x}\, e^{logx}$ iv)- e^{logx}

19) If $f'(a)$ exists, then $\lim_{h\to 0} \frac{f(a+h)- f(a-h)}{h}$ is equal to

(i) $\frac{f'(a)}{2}$ (ii)f'(a) (iii) $2f'(a)$ (iv) none of these

20) Let $f(x) = [x], x>0$ where $[x]$ denotes the integral part of x

I. f(x) is continuous for all integral value of x.

II. f(x) is not differentiable for all positive integral values of x.

III) $\lim_{x\to 2} f(x)$ does not exist .

Which of the above statements is correct?

i) I and II only ii) II and III only iii) I and III only iv) I,II and III

21) The function f(x) = |x+2| is not differentiable at

i) x=2 ii) x=-2 iii) x=-1 iv) x = 1

22) which of the following function is continuous but not differentiable

i) sin x ii) cos x iii) x+1 iv) |x|

23) Function f(x) = |x| is defined on [-2,2]. The points at which f is differentiable are

i) -1,0 ii) 1,0,1 iii) -1,0,1,2 iv) None of these

24. The function f(x) = x|x| (where x is real number) is

i) differentiable ii) differentiable except at x =0 iii) differentiable except at x=-1 iv) none of these

Answers

1. iv 2. iii 3. iii 4.false 5. ii 6.iv 7.ii 8.iv 9.iv 10.ii 11.iii 12. iii 13.i 14.ii 15.i 16.ii 17.i 18.iii 19.iii 20.ii 21. Ii 22. iv 23 iv 24 i

CHAPTER 6

APPLICATION OF DERIVATIVES

KEY POINTS

Concept or Formulas to Remember:

A. Derivatives As a Rate of change :

1. $\frac{dy}{dx}$ represents the rate of change of y with respect to x.
2. *Velocity*: Velocity is the rate of change of displacement(s) with respect to the time(t).

   so, Velocity = $\frac{ds}{dt}$.
3. *Acceleration:* Acceleration is the rate of change of displacement (s) with rtespect to time t.

   So, Acceleration = $\frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$
4. Two variables x and y are varying with respect to another variable t,

   i.e. x = f (t), y = g(t), then $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

B. Increasing and Decreasing Function:

1. Definition: Increasing and Decreasing Function in an open interval (a,b)

Let f be a real valued function and (a,b) be an open interval contained in the domain of f. Then a function is said to be

i) Increasing on (a,b) if for all $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)\ \forall\ x_1, x_2 \in (a,b)$

ii) Strictly Increasing on (a,b) if for all $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a,b)$

iii) Decreasing on (a,b) if for all $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)\ \forall\ x_1, x_2 \in (a,b)$

iv) Strictly Decreasing on (a,b) if for all $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in (a,b)$

2. Definition: Increasing and Decresing Function at a point

Let c be a point in the domain of the real valued function f , then f is increasing or strictly increasing or decreasing or strictly decreasing at c if there exists an

open interval containing c where f is increasing or strictly increasing or decreasing or strictly decreasing in that interval.

i.e. a function f is increasing at x = c if there exist an interval I = (c-h , c+h) such that $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)\ \forall\ x_1, x_2 \in I$

C. Tangent and Normals :

1. Slop of the tangent to a curve at a point

Let y= f(x) be a continuous curve and let $P(x_1 , y_1)$ be any point on the curve. Then slope of the tangent to a curve at a point $P(x_1, y_1) = \left(\frac{dy}{dx}\right)_{\text{at P}}$.

2. if the tangent is parallel to x axis (i.e. horizontal tangent), then the slope of the tangent at the point = 0 i.e., $\frac{dy}{dx} = 0$ at that point.
3. If the tangent is perpendicular to x-axis (i.e., vertical tangent), the slope of the tangent is undefined i.e., $\frac{dx}{dy} = 0$
4. Equation of the tangent to the curve y = f(x) at P (x_1, y_1)

The equation of the tangent to the curve y = f(x) at $P(x_1,y_1)$ is

$y-y_1 = \left(\frac{dy}{dx}\right)_{\text{at P}} (x-x_1)$

5. If the tangent to the curve y = f(x) makes an angle θ with x-axis in the positive direction, then $\frac{dy}{dx} = slope\ of\ the\ tangent = \tan\theta$
6. Normal To a Curve : Normal to a curve at appoint P is a line perpendicular to the tangent to the curve at that point.
7. Slope of the normal at a point : Slope of the normal at a point $P(x_1, y_1) = \frac{-1}{Slope\ of\ the\ tangent\ at\ P} = \frac{-1}{\left(\frac{dy}{dx}\right)atP} = -\left(\frac{dx}{dy}\right)_{\text{at P}}$.
8. Equation of the normal to the curve at P (x_1,y_1) is

   $y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)atP} (x - x_1)$.
9. If $\left(\frac{dy}{dx}\right)_{\text{at P}} = 0$ then the tangent is parallel to x-axis and the equation of the tangent is $y - y_1 = 0$.

10. If $(\frac{dy}{dx})_{at\ P}$ is undefined, then the tangent is parallel to y-axis and the equation of the tangent is $x - x_1 = 0$.

11. Algorithm for finding the equations of the tangent and normal to the curve y = f(x) at a point

    i) Obtain the point of contact (if not given)

    ii) Find $\frac{dy}{dx}$ from y = f(x)

    iii) Find $\frac{dy}{dx}$ at that point.

    iv) Find the equation of the tangent and normal using the formula.

12. The angle of intersection of two curves

    The angle of intersection between two curvesis defined as the angle between the tangent to the curves at there point of intersection. If m_1 , m_2 are the slopes of the tangent the curve at the point of intersection, then the acute angle between them is given by

    $\tan\theta = \left|\frac{m1-m2}{1+m1m2}\right|$

13. Orthogonal curves : if the angle of intersection of two curvesis a right angle, then the two curves cut orthogonally and the curves are orthogonal curves. If the curves cut orthogonally, $m_1m_2 = -1$

D. Maxima and Minima:

1. Maximum and minimum value of a function in its domain:

Let f(x) be a real valued function defined on the interval I, then

i) f(x) is said to have a maximum value in I, if there exist a point c in I such that $f(c) \geq f(x)$,$\forall$ $x \in I$. the number f(c) is called maximum value of f(x) in I.

ii) f(x) is said to have a minimum value in I, if there exist a point c in I such that $f(c) \leq f(x)$,$\forall$ $x \in I$. the number f(c) is called minimum value of f(x) in I.

iii) f(x) is said to have an extreme value in I, if there exist a point c in I such that f(c) is either a maximum value or a minimum value of f(x) in I.

2. First derivative test :

Let f be a real valued function.

i) Find the solution of f '(x) = 0. Let these values are a,b,c,….

ii) If f '(x) > 0 for x< a if f ' (x) < 0 for x > a , then function is said to attain local maximum at x = a and f(a) is the local maximum value at x = a.

i.e., f ' (x) changes from positive to negative as x increases through a.

iii) If f '(x) < 0 for x< a if f ' (x) > 0 for x > a , then function is said to attain local maximum at x = a and f(a) is the local minimum value at x = a.

i.e., f '(x) changes from negative to positive as x increases through a.

3. Second derivative test :

Let f(x) be a real valued function defined in an interval I and c ∈ I. let f(x) is twice differentiable at c. Then –

i) x = c is a point of local maximum if f ' (c) = 0 and f " (c) < 0. The value of f(c) is a local maximum value of f(x)

ii) x = c is a point of local minimum if f ' (c) = 0 and f " (c) > 0. The value of f(c) is a local minimum value of f(x)

iii) test fails if f '(c) =0 and f "(c) = 0. In this case we go back to the first derivative test and check whether c is a point of local maximum or local minimum.

E. **Approximation:**

Let y = f(x) be a function. Corresponding to small change △x in x, △y be the change in y.Then y + △y = f (x+△x) ⇒ △y = f (x+△x) - f(x)

We define the following :

i) the differential of x, denoted by dx is defined as dx = △x

ii) The differential of y, denoted by dy is defined as dy = f ' (x) dx or dy= ($\frac{dy}{dx}$) △x.

iii) In case dx = △x is relatively small when compared with x, dy is good approximation of △y

And we denote dy = △y

THREE LEVELS OF GRADED QUESTIONS

LEVEL I

1 An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

2 The total cost C(x)in Rupees associated with the production of x units of an item is given by $C(x)= 0.007x^3-0.003x^2+15x+4000$. Find the marginal cost when 17 units are produced.

3 The total revenue in Rupees received from the sale of x units of a product is given by $R(x)= 13x^2+26x+15$. Find the marginal revenue when x = 7

Level II

1 A balloon which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increasses when the radius is 15 cm.

2. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?.

3 The length x of a rectangle is decreasing at the rate of 5cm/ min. and the width y is increasing at the rate of 4 cm / min.When x = 8 cm and y= 6 cm, Find the rates of changes of a) the perimeter, and b) the area of the rectangle.

Level I (Increasing & decreasing functions)

1 Determine for which values of x , the function $y= x^4 - \frac{4x}{3}$ is increasing and for which values, it is decreasing.

2 Prove that the function f(x)=tan x – 4x is strictly decreasing on $\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$

LEVEL II

1 Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

2. Find the intervals in which the function f given by $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

LEVEL III

1 Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is increasing function of θ *in* $\left[0, \frac{\pi}{2}\right]$

2 Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1,$ is an increasing function of x throughout its domain.

3 Find the values of x for which $y=[x(x-2)]^2$ is an increasing function.

4 Find intervals in which the following function

f(x) = $\frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is

a) Strictly increasing.
b) Strictly decreasing.

5 On which of the following intervals is the function f given by f(x) = x^{100}+sin x -1 strictly decreasing ?

(0,1) b) ($\frac{\pi}{2}, \pi$) c) (0, $\frac{\pi}{2}$) d) None of these**.**

2 Find the points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are

(i) parallel to x-axis (ii) parallel to – y axis

3 Find the equation of the tangent line to the curve $y= x^2-2x+7$ which is
a) Parallel to the line 2x-y+9=0
b) perpendicular to the line 5y-15x =13

4 Find the equations of the tangent and normal to the curve $x = 1 - \cos\theta, y = \theta - \sin\theta$ at $= \frac{\pi}{4}$. (Mar 2010)

5 Prove that the curves $x= y^2$ and xy=k cut at right angle if $8k^2=1$

6 At what points will the tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel

to x- axis? Also, find the equations of tangents to the curve at those points.(Mar2008)

7 Prove that x/a + y/b = 1 is touching the curve $y = be^{-x/a}$ where the curve cuts the Y- axis.

Tangents & Normals & Maxima Minima

1 Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ *at* (1,1)

2 Find the equation of the tangent to the curve y=√(5x -3) -2 which is parallel to the line 4x-2y +3 =0

3 Show that the curve $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0,0)

4 Find the condition for the curves $x^2/a^2 - y^2/b^2 = 1$ and $xy = c^2$ tointersect orthogonally.

5 Find the equations of the tangents to the curve $3x^2 - y^2 = 8$ which passes through the point (4/3, 0).(Mar 2013)

6 Find the equations of tangent and normal to the curve $y=\frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis .(Mar 2013)

8. Show that the right circular cylinder of given surface and maximum volume is such that it height is equal to the diameter of the base.

9. Show that of all the rectangles inscribed ina given fixed circle, the square has the maximum area.

10. Of all the closed cylindrical cans (right circular) , of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area?

11. A rectangular sheet of tin 45cm by 24cm is to be made into a box whithout top, by cutting off square from each corner and folding up the flaps . What should be the side of the square to be cut off so that the volume of the box is maximum?

1 MARKS QUESTIONS

1. An edge of a variable cube is increasing at the rate of 5 cm per second. How fast is the volume increasing when the side is 15 cm?
2. Find the slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at $t = 2$.
3. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 4 cm.
4. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.
5. Show that $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.
6. Find the slope of normal to the curve $x^2 + y^2 = 8$ at (2,2)
7. Show that $f(x) = x^3 - 3x + 4x$, $x \in R$ is strictly increasing on R.
8. Find the slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$.
9. Find the slope of normal to the curve $x = \cos t$, $y = \sin t$ at $t = \frac{\pi}{4}$.
10. A balloon, which always remains spherical, has a variable diameter $\frac{2}{3}(3x + 1)$. Find the rate of change of its volume with respect to x.
11. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?
12. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?
13. Show that the function given by $f(x) = 3x + 17$ is strictly increasing on R
14. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.
15. Prove that the logarithmic function is strictly increasing on $(0, \infty)$
16. Find the slope of the tangent to the curve $y = 3x^3 - 4x$ at $x = 10$
17. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose X- Coordinate is 3.
18. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to x axis.
19. Prove that the following functions do not have maxima or minima :
    $f(x) = e^x$
20. What is the maximum value of the function $\sin x + \cos x$?

ANSWERS

1. $\frac{dx}{dt}$ = 3375 cm^3/s. 2. $\frac{dy}{dx} = \frac{6}{7}$. 3. $\frac{dA}{dr}$ = 8π cm^2/cm 4. slope = 764

5. f'(x) = 3x^2- 6x + 6 = 3(x^2 - 2x +1) = 3(x – 1)^2 ≥ 0 f is increasing on R.

6. slope of normal = - $\frac{1}{-1}$ = 1.

7.F'(x) = 3x^2 – 6x + 4 = 3(x-1)^2 + 1 > 0 for all x ∈ R . increasing on R.

8. y' = 4x + 3 cosx , at x= 0 , y' = 3, slope of normal to the curve = - $\frac{1}{3}$.

9. $\frac{dy}{dx} = \frac{cost}{-sint}$ = - cot t. At t =$\frac{\pi}{4}$, $\frac{dy}{dx}$ = -1 slope of normal is - $\frac{1}{\frac{dy}{dx}}$ = 1.

10.$\frac{27\pi}{8}$(2x + 1)^2 11. 900 cm^3/s 12.60π cm^2/s 16. 896

17. 24 18. (3,-20) and (-1,12) 20. $\sqrt{2}$

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Chapter -7

INTEGRALS

KEY POINTS

- Let $\frac{d}{dx}$F(x) = f(x). Then we write$\int f(x)dx$ = F(x) +C. These integrals are called indefinite integrals or general integral; C is called integral constant.
- Some properties of integrals:
  (i) $\int(f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$.
  (ii) $\int kf(x)dx = k\int f(x)dx$, k is any scalar.
- Some standard integrals:
  (i) $\int cosec\ x\ dx = \log(\text{cosec}\ x - \cot x) + C$
  (ii) $\int \sec x\ dx = \log(\sec x + \tan x) + C$
  (iii) $\int \frac{dx}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
  (iv) $\int \frac{dx}{a^2-x^2}dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + C$
  (v) $\int \frac{dx}{x^2-a^2}dx = \frac{1}{2a}\log\left(\frac{x+a}{x-a}\right) + C$

(vi) $\int \frac{dx}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a} + C$

(vii) $\int \frac{dx}{\sqrt{a^2+x^2}} dx = \log(x + \sqrt{a^2+x^2}) + C$

(viii) $\int \frac{dx}{\sqrt{x^2-a^2}} dx = \log(x + \sqrt{x^2-a^2}) + C$

(ix) $\int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2-a^2}) + C$

(x) $\int \sqrt{x^2+a^2}\ dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2+a^2}) + C$

(xi) $\int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

- Integration by parts: $\int (I\ .\ II)dx = I\ .\int II\ dx - \int (\frac{d}{dx}(I).\int II\ dx)\ dx$
- Some properties of definite integrals:

(i) $\int_a^b f(x)dx = \int_a^b f(y)dy$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(iv) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(v) $\int_{-a}^a f(x)dx = \begin{cases} \int_0^{2a} f(x)dx & if\ f(-x) = f(x) \\ 0 & if\ f(-x) = -f(x) \end{cases}$

LEVEL-1

1. $\int \sec x(\sec x + \tan x)dx$

2. $\int \frac{x^3 - x^2 + x - 1}{x-1}\ dx$

3 $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

4 $\int \frac{1}{\sin x \cos x} dx$

5. $\int \frac{\sec^2(\log x)}{x} dx$

6. $\int \sin^3 x\, dx$

7. $\int \cos x \cos 2x \cos 3x\, dx$

8. $\int \frac{dx}{\sqrt{4x^2-9}}$

LEVEL :1

9. $\int \frac{dx}{x^2+2x+10}$

10. $\int \frac{dx}{9x^2+12x+13}$

11. $\int \frac{2x+1}{(x+1)(x-1)}dx$

12. $\int x\sec^2 x\,dx$

13. $\int e^x(\tan x+\log\sec x)dx$

14. $\int \sqrt{4+x^2}\,dx$

15. $\int \sqrt{1-4x^2}\,dx$

16. $\int (9x+\sin x+\cos eecx)dx$

17. $\int_0^2 x\sqrt{x+2}\,dx$

18. $Evaluate\int_0^2 (x+2)dx\,as\,a\lim it\,of\,a\,sum$

19. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}dx$

22. $\int_{-1}^{1} 5x^4\sqrt{x^5+1}dx$

23. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$

24. $\int \sin^{-1}(\cos x)dx$

25. $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}dx, \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$

LEVEL -2

Evaluate

1 $\int \frac{x^2+5x-1}{\sqrt{x}}\,dx$

2 $\int \cos(\pi x+1)dx$

3 $\int \cos x \cos 2x \cos 5x\, dx$

4 $\int sin^2 3x dx$

5 $\int \frac{dx}{x^2+4x+5}$

6 $\int \frac{dx}{\sqrt{5-x^2-4x}}$

7 $\int \frac{2+3x^2}{x^2(1+x^2)}dx$

8 $\int \frac{\sqrt{1-x^2}+\sqrt{1+x^2}}{\sqrt{1-x^4}}dx$

9 $\int \frac{dx}{1+e^x}$

10 $\int \frac{x^2+3}{\sqrt{(2x-3)^3}}$

11 $\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\frac{x^2+1}{x}}$

12 $\int x^3 \log x\, dx$

13 $\int \cos(\log x)dx$

14 $\int x \log\left(1+\frac{1}{x}\right)dx$

15 $\int \sin x \log(\tan x)dx$

16 $\int \frac{2x^2-3x+3}{x^3-2x^2+x}dx$

17 $\int \frac{dx}{\sin x(2cos^2 x-1)}dx$

18 $\int_1^2 |1-x|dx$

19 $\int_0^{\pi} \frac{\sqrt{1+cos2x}}{\sqrt{2}}dx$

20 $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - cos^3 x}\ dx$

21 $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$

22 $\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{cos^4 x + sin^4 x} dx$

23 $\int_0^{\frac{\pi}{2}} x^2 \sin x dx$

24 $\int_0^4 \{|x| + |x-2| + |x-4|\} dx$

25 $\int \frac{dx}{(x+2)(x-1)(x+1)}$

26 $\int \frac{2x^2-3x+3}{x^3-2x^2+x} dx$

ONE MARK/ TWO MARKS QUESTIONS

1 .Evaluate $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x . \cos^2 x} dx$. (DELHI 2014C)

2. Evaluate $\int \frac{\sin^6 x}{\cos^8 x} dx$ (AICBSE 2014 C)

3. Evaluate $\int \frac{dx}{\sin^2 x . \cos^2 x}$

4. Evaluate $\int \cos^{-1}(\sin x) dx$ (DELHI 2014)

5. Write anti derivative of $(3\sqrt{x} + \frac{1}{\sqrt{x}})$.

6. Evaluate $\int (1-x)\sqrt{x}\, dx$ (DELHI 2012)

7. Evaluate $\int e^x (1 + \tan x) \sec x\, dx$ (AICBSE 2012)

8. Evaluate $\int \frac{2}{1+\cos 2x} dx$ (FOREIGN 2012)

9. Evaluate $\int \frac{x+\cos 6x}{3x^2+\sin 6x} dx$

10. Write the value of $\int \frac{dx}{x^2+16}$ (DELHI 2011)

11. Write the value of $\int \frac{\sec^2 x}{cosec^2 x}\,dx$ (DELHI 2012 C)

12. Evaluate $\int \frac{2-3\sin x}{\cos^2 x}\,dx$

13. Evaluate $\int \frac{dx}{\sqrt{1-x^2}}\,dx$ (AISSSE 2011)

14. Evaluate : $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

15. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$

6. Evaluate: $\int_0^1 \frac{1}{\sqrt{2x+3}}\,dx$

17. Find the value of the integral: $\int_{-1}^{1} x^{17}\cos^4 x\,dx$. (DELHI 2014 C)

18. Find the value of k for which $\int_0^1 (3x^2+2x+k)dx$=0

19. Evaluate:-$\int_0^1 x(1-x)^{89}\,dx$ (DELHI 2011)

20. $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then find the value of a.

ANSWERS

1. $(\tan x + \cot x) + C$
2. $\left(\frac{1}{7}tan^7 x\right) + C$
3. $(\tan x - \cot x) + C$
4. $\frac{\pi}{2}x - \frac{x^2}{2} + C$
5. $2\sqrt{x}(x+1) + C$
6. $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$
7. $e^x \sec x + C$
8. $\tan x + C$
9. $\frac{1}{6}\log|3x^2 + \sin 6x| + C$. 10 $\frac{1}{4}\tan^{-1}\frac{x}{4} + C$
10. $\frac{1}{4}\tan^{-1}\frac{x}{4} + C$
11. $(\tan x - x) + C$.
12. $(2\tan x - 3\sec x) + C$.
13. $\sin^{-1} x + C$.
14. $\frac{\pi}{4}$
15. $\frac{\pi}{4}$
16. $\sqrt{5} - \sqrt{3}$
17. 0
18. K = -2
19. $\frac{1}{8190}$
20. a = $\frac{1}{2}$

CHAPTER 8

Application of Integrals

LEARNING OBJECTIVES/OUTCOMES

1.Area under the curve y=f(x)and the x-axis and the ordinates at x=a and at x=b is $\int_a^b ydx$.

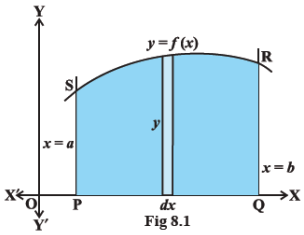
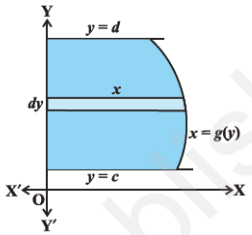
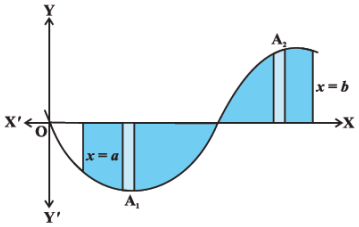


Fig 8.1

2.Area under the curve x=f(y)and the x-axis and the ordinates at y=c and at y= d is $\int_c^d xdy$



3.The area bounded by the curve y=f(x) and x-axis and the ordinates x=a and x=b is given by $A_1 + A_2 = A$



4.Area under two curves

If y=f(x) , y= g(x) where f(x) ≥ g(x) in the [a,b] such that the point of intersection of these two curves are given by x=a and x=b obtained by taking common values of y from given equation of two curves then the area between the curves is given by $A=\int_a^b [f(x) - g(x)]dx$

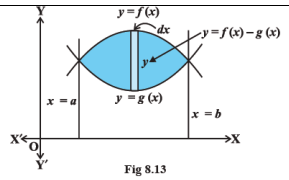


Fig 8.13

$dA = [f(x) - g(x)]\,dx$, and the total area A can be taken as

$$A = \int_a^b [f(x) - g(x)]\,dx$$

5.If f(x) ≥ g(x) [a,c] and f(x) ≤ g(x) in [c,b] where a < c<b , then area of the region bounded by the curves can be given by

Total Area = Area of the region ACBDA + Area of the region BPRQB

$$= \int_a^c [f(x) - g(x)]\,dx + \int_c^b [g(x) - f(x)]\,dx$$

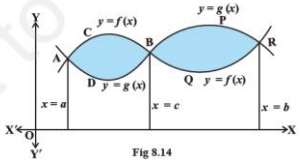
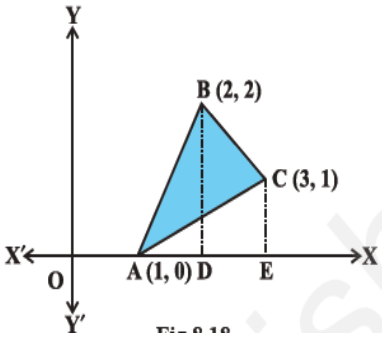


Fig 8.14

6.Area of a triangle when the coordinates of the vertices or equations of sides are given.



STEPS FOR FINDING THE AREA USING INTEGRATION

STEPS

- DRAW THE DIAGRAM
- MAKE A SHADED REGION
- FIND INTERSECTION POINTS
- IDENTIFY THE LIMITS
- WRITE THE INTEGRAL(S) FOR THE REGION
- EVALUATE THE INTEGRAL
- THE VALUE SHOULD BE POSITIVE

THREE LEVELS OF GRADED QUESTIONS

LEVEL I

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the x - axis.

2. Find the area of the region bounded by $y = x^2$ and the lines $y = 2$ and $y = 4$ and the y- axis.

3. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

4. Find the area of the region bounded by the parabola $y=x^2$ and $y=x$

5. Find the area of the region bounded by the curve $y = x$ between $x = -1$ and $x=1$.

6. Find the area of the region bounded by the curve $x^2=4y$ and the line $x=4y-2$

7.Find the area of the region bounded by the curve $y^2= 4x$ and the line $x= 3$

8 .Find the area of the region lying in the first quadrant and bounded by $y=4x^2$, $x=0$,$y=1$ and $y=4$

9. Find the area of the region enclosed by the parabola $x^2 = y$, and the line $y = x+2$ and the x axis

10. Find the area lying in the first quadrant and bounded by the circle $x^2+ y^2=4$ and the lines $x=0$ and $x=2$.

11. Find the area of the region bounded by the curve $y = \frac{1}{x}, x -$ axis and between $x = 1, x = 4$

12 Find the area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ *and* $x = 3$

13 Find the area of the region bounded by the curve $y = cosx,$ between $x = 0$ *and* $x = \pi$

14 Find the area bounded by the curve $x^2 + y^2 = 4$ in first quadrant

LEVEL 2

1. Find the area of the region bounded by the parabola $y = x^2$ and $y^2 = x$

2. Find the area of the region bounded by the parabola $y= x^2$and $y = |x|$

3 Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$

4. Using Integration find the area of the region bounded by the triangle whose vertices
are (1,0), (2,2) and (3,1)

5. Using integration find the area of the triangular region whose sides have the equations y = 2x + 1, y = 3x + 1 and x = 4

6. Find the area bounded by the curve y = sin x between x = 0 and x =2 π

7. Find the area of the smaller region enclosed by the circle $x^2 + y^2 = 4$ and the line

$$x + y = 2$$

8. Find the area of the parabola $y^2=4ax$ bounded by the latus rectum.

9. Find the area of the region bounded by the line y=3x+2, the x-axis and the ordinates x=-1 and x=1

10. Find the area of the region included between $4y=3x^2$, and the line 3x-2y+12=0 .

11. Find area of the region $\{ (x, y) : y \leq x^2 + 1, y \leq x+1, 0 \leq x \leq 2\}$

12. Draw the rough sketch and find the area of the region bounded by two parabolas

$4y^2 = 9x$ and $3x^2 = 16y$ by using the method of integration.

13. Find the area enclosed by the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$

14 Find the smaller of the two areas enclosed between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the line $bx + ay = ab$

15. Find the smaller of two areas bounded by the curve $y = |x|$ and $x^2 + y^2 = 8$

LEVEL 3

1. Sketch the region bounded by the curves $y= \sqrt{5 - x^2}$ and $y= |x - 1|$ and find its area.

2. Make a rough sketch of the region given below and find its area using integration $\{ (x, y); 0\leq y\leq x^2, 0 \leq y \leq 2x+ 3, 0\leq x\leq 3 \}$

3. Find the ratio of the area into which the curve $y^2=6x$ divides the region bounded by $x^2+y^2 =4$ and $(x+2)^2+ y^2= 4$ using integration

1. Sketch the graph of $y = |x + 1|$. Evaluate $\int_{-3}^{1} |x + 1| dx$

6.Find the area lying above x-axis and included between the circles $x^2+y^2=8x$ and the parabola $y^2=4x$

7. Find the area of the region between the two curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$

8. Prove that the curves $y^2=4x$ and $x^2=4y$ divide the area of the square bounded by $x=0$, $x=4$, $y=4$ and $y=0$ into three equal parts.

9. Find the area of the region enclosed between two circles $x^2+y^2=9$ and $(x-3)^2+y^2=9$

10 Using integration , find the area of the region bounded by the triangle whose vertices are $(-2,2)(0,5)and\ (3,2)$

11 Find the area bounded by curves $y=|x|-1$ and $y=1-|x|$

12 Find the area bounded by the curves $y=|x-1|$and $y=3-|x|$

13 Find the area bounded by curve $y=x|x|$ and x -axis and the ordinates $x=-1$ and $x=1$

14 Find the area of the region $\{(x,y): 0\leq y\leq x^2+3, 0\leq y\leq 2x+3, 0\leq x\leq 3\}$

CHAPTER 9

DIFFERENTIAL EQUATIONS

Key Concepts:

1. The equation containing derivatives of any order, dependent variable and independent variable is called DIFFERENTIAL EQUATION.
2. The highest order of derivative is called order of DIFFERENTIAL EQUATION.
3. The degree of highest order of derivative is degree of DIFFERENTIAL EQUATION after removing radical signs.(it should be in the form of polynomials)
4. The order of the differential equation will be equal to the number of arbitrary consonants in the primitive.
5. To solve differential equation in the variable separation method separate the variables and integrate.

6. To solve equation of homogeneous type put y=v x (or x = v y)
7. To solve linear differential equation (L.D.E) find I.F.
8. I.F of $\frac{dy}{dx} + py = q$ is $e^{\int pdx}$

THREE LEVELS OF GRADED QUESTIONS

LEVEL 1

1. Find the order and degree of the following differential equations:
   a. $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0.$
   b. $\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}.$
   c. $\left(\frac{d^2y}{dx^2}\right)^3 + \sin\left(\frac{dy}{dx}\right) = 0.$
   d. $(y''')^2$ - $2(y'')$ -y' =-1.

2. Verify the given functions is the solution of the corresponding differential equation;
   a. y = $x^2 + 2x + c : y' - 2x - 2 = 0.$
   b. $y = \cos x + C \ : \ y' + \sin x = 0.$
3. Form the differential equation representing the family of curves $y^2 - 2ay + x^2 = a^2$, where "a" is arbitrary constant.
4. Form the differential equation representing y =e^{2x}(a + bx).
5. Solve the differential equation: $(e^x + e^{-x})\,dy = (e^x - e^{-x})\,dx$.
6. Solve the differential equation $ydx - x\,dy = y^2$
7. Solve the differential equation: $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0.$
8. Solve: $(x^2 - y^2)\,dx + 2xy\,dy = 0$ given that y=1 when x=1.
9. Solve: $\frac{dy}{dx} = \frac{(2y-x)}{(2y+x)}$ $if\ y = 1\ when\ x = 1.$
10. Solve: $(x^3 + y^3)\,dy - x^2y\,dx = 0.$
11. Solve: $y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0.$
12. Solve: $\frac{dy}{dx} + y = \cos x - \sin x.$
13. Solve: $\cos^2 x\frac{dy}{dx} + y = \tan x.$
14. Solve: $x\frac{dy}{dx} + y = x\log x.$
15. Find the general solution of the differential equation: $y\,dx - (x + 2y^2)\,dy = 0.$

16. Solve: $(x+y)\frac{dy}{dx}=1$.

17 Solve :$\frac{dy}{dx}=-\frac{y}{x}$

18 Solve : $(x+1)\frac{dy}{dx}=x(y^2+1)$

19 Solve :$(y-1)x\frac{dy}{dx}=y(1+x)$

20 Solve : $\frac{dy}{dx}+y=e^{-x}$

21 Solve: $\frac{dy}{dx}+y=e^{e^x}$

22 Find the differential equation of family straight lines passing through origin

23 Find the differential equation of all non- horizontal lines in a plane

24 The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$

25 Find the order and degree of the differential equations

i)$\left(1-\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}=k\frac{d^2y}{dx^2}$

ii)$y'''+y^2+e^{y'}=0$

iii)$3x\frac{dy}{dx}+\frac{5}{\frac{dy}{dx}}=y^3$

26 Verify $xy=\log y+c$ is the solution of the differential $(xy-1)\frac{dy}{dx}+y^2=0$

27 Find the product of the order and degree of the following differential equation:

$$x\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2+y^2=0$$

28 Write a differential equation for $y=Acos\alpha x+Bsin\alpha x$, where A and B are arbitrary constants.

LEVEL 2

29. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

30. Find the particular solution of the differential equation: $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$, given that y=1 when x=0.

31. Solve: $\frac{dy}{dx}=\frac{1-cosx}{1+cosx}$

32. Solve: $y\log y\,dx-x\,dy=0$.

33. Find the equation of the curve passing through the point (0,0) whose differential equation is $\frac{dy}{dx}=e^xsinx$.

34. Solve: $(x+y)^2\frac{dy}{dx}=a^2$.

35. Solve: $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, y=0 when x=1.

36. Solve: $\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} y\,dx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} x\,dy.$

37. Solve: $2xy + y^2 - 2x^2\frac{dy}{dx} = 0, y = 2\ when\ x = 1.$

38. Show that the differential equation $2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous. Find the particular solution given that x=0 when y=1.

39. Solve: $\frac{dy}{dx} - y = \cos x.$

40. Solve: $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x.$

41. Solve: $\frac{dy}{dx} - 3y\cot x = \sin 2x; y = 2\ when\ x = \frac{\pi}{2}.$

43 Solve : $(x+1)\frac{dy}{dx} - 2y = x(x+1)^2$

44 Solve :$secx\frac{dy}{dx} = y + sinx$

45 Solve :$x\frac{dy}{dx} + 3y = \frac{sinx}{x^2}$

46 Find order of the differential equation of all circles of given radius a

47 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$.

48 Solve the differential equation , given that $y = 0$, when $x = \frac{\pi}{4}$:

$sin2x\frac{dy}{dx} - y = tanx.$

49 Show that the general solution of the differential equation $\frac{d^2y}{dx^2} - y = 0$ is $y = c_1e^x + c_2e^{-x}$

50 Solve $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{x}{y}\right), x \neq 0\ and\ x = 1, y = \frac{\pi}{2}$

LEVEL 3

1. Find the order and degree of the following differential equation:

(a) $\frac{d^4y}{dx^4} + sinx = \left(\frac{d^2y}{dx^2}\right)^5$. (b) $\frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0.$

2. Write the integrating factor of the following differential equation:

a. $\frac{dy}{dx} + y\cos x = \sin x$

b. $x^2\frac{dy}{dx} + y = x^4$

c. $\frac{dy}{dx} + y\tan x = \sec x$

d. $\frac{dy}{dx} + \frac{y}{1+x^2} = \sin x$

3. Solve: $\cos^3 x\frac{dy}{dx} + \cos x = \sin x.$

Solve: $y\,dx + (x - y^3)\,dy = 0$.

4. 48. Solve: $ye^y\,dx = (y^3 + 2xe^y)\,dy$.
5. 49. Solve: $\frac{dy}{dx} = y\sin^3 x.\cos^3 x + xye^x$.
6. Verify that

   $y = Sin(Sinx) is\ a\ solution\ of\ differential\ equation \frac{d^2y}{dx^2} + tanx.\frac{dy}{dx} + y\cos^2 x = 0.$
7. Solve: $\left(y\sin\left(\frac{x}{y}\right)\right)dx = \left(x\sin\left(\frac{x}{y}\right) - y\right)dy$.
8. Solve: $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.
9. Solve :$\frac{dy}{dx} = \frac{xy}{x^2 - y^2}$
10. Solve : $x\frac{dy}{dx} = y(logy - logx + 1)$
11. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x} y(1) = \frac{\pi}{2}$
12. Solve: $\frac{dy}{dx} + ysecx = tanx$
13. Find order of the differential equation of all circles of given radius a
14. Find the equation of a curve passing through origin if the slope at any point is equal to the square of the difference of the abscissa and ordinate of the point.

Chapter 10

Vectors

1. Magnitude of a vector means length of a vector.

   If $\vec{a}$ = i, then $|\vec{a}|$ = 1, If $\vec{a}$ = i + j , then $|\vec{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}$ and

   If $\vec{a}$ = 2i + 3j + 4k, then $|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$.
2. Unit vector along a vector $\vec{a}$ is defined as $\hat{n} = \frac{\vec{a}}{|\vec{a}|}$.
3. Direction cosine of a vector $\vec{a}$ is the cosine of the angle made by the vector $\vec{a}$with the positive directions of x, y, z axis.
4. Triangle law of vectors: $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ where AB, BC, CA are the three sides of a triangle taken in order.
5. Position vector of a point P is defined as $\overrightarrow{OP}$, where O is the origin.
6. Vector joining two points A, B is defined as $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

7. Section formula :The position vector of the point R which divides P and Q internally in the ratio m:n is given by $\overrightarrow{OR} = \frac{m\vec{b}+n\vec{a}}{m+n}$. For the case of externally it is $\overrightarrow{OR} = \frac{m\vec{b}-n\vec{a}}{m-n}$
8. Scalar product of vectors $\vec{a}$and $\vec{b}$ is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where θ is the angle made by the vectors $\vec{a}$and $\vec{b}$.It is called scalar because the result in right hand side of the formula is a scalar quantity.
9. Scalar product of three vectors is defined as $\vec{a}.(\vec{b} \times \vec{c})$.
10. Volume of the parallelepiped =$\vec{a}.(\vec{b} \times \vec{c})$ cubic units.
11. If $\vec{a}.(\vec{b} \times \vec{c}) = 0$, then $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.
12. Vector product of two vectors is defined as $(\vec{a} \times \vec{b}) = |\vec{a}||\vec{b}| \sin\theta\hat{n}$.
    Where θ is the angle between the vectors $\vec{a}$and $\vec{b}$ and $\hat{n}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.
13. Vector product of vectors $\vec{b}$ and $\vec{c}$in determinant form

$\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ is $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

14. Area of a triangle= $\frac{1}{2}|\vec{a} \times \vec{b}|$ sq. units.

15.Area of a parallelogram= $|\vec{a} \times \vec{b}|$sq. units when $\vec{a}$*and* $\vec{b}$ are adjacent sides.

Other important concepts :

1. $|\vec{a}|^2 = \vec{a}.\vec{a}$
2. Parallelism of two vectors $\vec{a}$ *and* $\vec{b}$: $\vec{a} = k\vec{b}$, where k is any constant.
3. Collinearity of three vectors $\vec{a}$,$\vec{b}$*and* $\vec{c}$:
   $\vec{a} = \alpha\vec{b} + \beta\vec{c}$, where α *and* β are any scalor.
4. Perpendicularity of two vectors : $\vec{a}.\vec{b} = 0$
5. Parallelism of Parallelism of two vectors $\vec{a}$ *and* $\vec{b}$: $\vec{a} \times \vec{b} = 0$
6. Angle between two vectors : $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$; $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
7. Scalar triple product of vectors in determinant form for the vectors

$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$, $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ is

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Dot product	**Cross product**	**Scalar triple product**
$\vec{a}.\vec{b} = \lvert\vec{a}\rvert.\lvert\vec{b}\rvert\, cos\theta$	$\vec{a}X\vec{b} = \lvert\vec{a}\rvert.\lvert\vec{b}\rvert \sin\theta\, \hat{n}$ where $\hat{n}$ is perpendicular to both $\vec{a}, b$.	Notation is $[\vec{a}, b, \vec{c}]$
If $\perp$ $\vec{a}.\vec{b} = 0$	If parallel $\vec{a}X\vec{b} = 0$	If coplanar $[\vec{a}, b, \vec{c}] = 0$
$\vec{a}.\vec{a} = \lvert\vec{a}\rvert^2$ $(\vec{a}.\vec{b})^2 = (\vec{a})^2 + (\vec{b})^2 + 2\vec{a}.\vec{b}$	$\vec{a}X\vec{a} = 0$	$[\vec{a}, b, \vec{c}] = 0$ if any two vectors are equal.
If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ Then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ Projection of $\vec{a}$ on $\vec{b}$: $\frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert}$	If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ Then $\vec{a}X\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$	If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ Then $[\vec{a}, b, \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

THREE LEVELS OF GRADED QUESTIONS

Level 1

1. Find the angle between the vectors $\hat{\imath}$-2$\hat{\jmath}$+3$\hat{k}$ and 3 $\hat{\imath}$-2$\hat{\jmath}$+$\hat{k}$
2. Find projection of the vector$\hat{\imath} + 3\hat{\jmath}$+7$\hat{k}$ on the vector7$\hat{\imath}$-$\hat{\jmath}$+8$\hat{k}$
3. Evaluate the product $(3\overrightarrow{a}$- $5\vec{b}).(\,2\vec{a}\, +7\vec{b})$
4. Find the dr's of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$.
5. Find the area of the parallelogram whose adjacent sides are determined by the vectors$\overrightarrow{a} = \hat{\imath} - \hat{\jmath}$+3$\hat{k}$ and $\overrightarrow{b}$ =2$\hat{\imath} - 7\hat{\jmath}$+$\hat{k}$.
6. If P(1,5,4) and $\boldsymbol{Q}(4,1,-2)$ find the direction ratios of $\overrightarrow{\mathrm{PQ}}$.
7. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$and $\vec{\mathrm{b}} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$, find a unit vector in the direction $\vec{a} - \vec{b}$.
8. If $\lvert\vec{a}\rvert = \sqrt{3}$,$\lvert\vec{\mathrm{b}}\rvert = 2$ and $\vec{a}.\vec{b} = 3$, find the angle between $\vec{a}$ and $\vec{b}$.
9. What is the angle between $\vec{a}$ and $\vec{b}$ with the vectors of magnitude $\sqrt{3}$ and 2 respectively and$\vec{a}.\vec{b} = 3$.

10. Write the position vector of the point dividing the line segment joining the points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1:4, where $\vec{\boldsymbol{a}} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{\boldsymbol{b}} = -\hat{\imath} + \hat{\jmath} + \hat{k}$.
11. Given $\vec{\boldsymbol{a}}$ =$2\hat{\imath} - \hat{\jmath}$+$\hat{k}$ and $\vec{\boldsymbol{b}}$ =$\mu\hat{\jmath}$-$4\hat{k}$, find μ such that$\vec{\boldsymbol{a}}.\vec{\boldsymbol{b}}$ =3.
12. Find a vector in the direction of $\vec{a} = \hat{\imath} - 2\hat{\jmath}$ that has magnitude "7" units.
13. Find the value of λ for which the vectors $3\hat{\imath} + 2\hat{\jmath} - \hat{k} and\ \hat{\imath} - \hat{\jmath} + \lambda\hat{k}\ are\ perpendicular\ to\ each\ other$.
14. If vector$\vec{\boldsymbol{a}}$ = $\mu\hat{\imath}$ +$3\hat{\jmath}$,$|\vec{\boldsymbol{a}}|$ =5,find μ.
15. Vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$and $\hat{\imath} + \hat{\jmath} - 3\hat{k}$ act along the adjacent sides of a parallelogram. Find the angle between the diagonals of the parallelogram.
16. Show that the area of the parallelogram having diagonals $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ is $5\sqrt{3}$. Sq.units.
17. If the vertices A,B,C, of a triangle ABC are (1,2,3), (-1,0,0) , (0,1,2) respectively , then find area of triangle ABC .
18. If a unit vector "a" makes an angle $\frac{\pi}{3}$ with$\hat{\imath}$, $\frac{\pi}{3}$ with $\hat{\jmath}$ and an acute angle θ with$\hat{k}$, then find θ and hence the components of$\vec{a}$.
19. If vectors $\vec{a}\ ,\vec{b}\ and\ \vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = 0$and $|\vec{\boldsymbol{a}}| = 3$,$|\vec{\boldsymbol{b}}| = 5$ and $|\vec{\boldsymbol{c}}|$ = 7 , find the angle between $\vec{a}$ and $\vec{b}$.
20. If the vectors $a\hat{\imath} + a\hat{\jmath} + c\hat{k}, \hat{\imath} + \hat{k} and\ c\hat{\imath} + c\hat{\jmath} + b\hat{k}$ are coplanar show that $c^2 = ab$.
21. Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2,1) and terminal point B(-5,7).
22. Show that the points A(-2,3,5) , B(1,2,3) and C(7,0,-1) are collinear.
23. Find the unit vector in the direction of the resultant of $\vec{a} = 2\hat{\imath} - \hat{\jmath}, \vec{b} = 3\hat{\imath} + 2\hat{\jmath}$

Level 2

1. If $\vec{a}.\vec{a}$ = 0 and $\vec{a}.\vec{b}$ = 0 then what can be concluded about the vector $\vec{b}$?

2. If $\vec{a}$ = $\hat{\imath}$+ $\hat{\jmath}$+ $\hat{k}$, $\vec{b}$= 2$\hat{\imath}$-$\hat{\jmath}$+3$\hat{k}$ and c= $\hat{\imath}$-2$\hat{\jmath}$+$\hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b}$ +3$\vec{c}$.or given vector $\vec{a}$= 2$\hat{\imath}$-$\hat{\jmath}$ +2$\hat{k}$ and $\vec{b}$ = -$\hat{\imath}$ +$\hat{\jmath}$-$\hat{k}$, find the unit vector in the direction of the vector $\vec{a}$+$\vec{b}$.

3. Let $\vec{a}$,$\vec{b}$, and $\vec{c}$ be three vectors such that $|\vec{a}|$ = 3 , $|\vec{b}|$ = 4 and $|c|$ = 5 and each one of them being perpendicular to the sum of the other two , find $|\vec{a} + \vec{b} + \vec{c}|$.

5.Show that $|\vec{a}|\vec{b}$ + $|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}$ - $|\vec{b}|\vec{a}$, for any two non- zero vectors $\vec{a}$and $\vec{b}$.

6. If a,b,c are lengths of the opposite sides respectively to the angles A,B,C of a triangle ABC, Prove that

i) $\cos A = \frac{b^2+c^2-a^2}{2bc}$ii) $a = b\cos C + c\cos B$.

7. Find the volume of the parallelepiped whose adjacent sides are represented by $\vec{a}, \vec{b}, \vec{c}$ *where* $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}, \vec{b} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}, \vec{c} = -4\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

Level 3

1. If $\vec{a}$ =$\vec{b}$ +$\vec{c}$, then is it true that $|\vec{a}|$ = $|\vec{b}|$ + $|\vec{c}|$? Justify your answer.

2. Write down a unit vector in XY plane making an angle of 30^0 in the positive direction of X- axis.

3.If$\vec{a} = \vec{\imath} + \vec{\jmath}, \vec{b} = \vec{\jmath} + \vec{k}$ and $\vec{c} = \vec{k} + \vec{\imath}$, find the unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$

4.Let$\vec{a}$ *and* $\vec{b}$ *be two vectors such that* $|\vec{a}| = 3$ *and* $|\vec{b}| = \frac{\sqrt{2}}{3}$ *and* $\vec{a} \times \vec{b}$ *is a unit vector.Then what is the angle between* $\vec{a}$ *and* $\vec{b}$.

5. Write the value of $\vec{\imath}.(\vec{\jmath} \times \vec{k}) + \vec{\jmath}.(\vec{k} \times \vec{\imath}) + \vec{k}.(\vec{\jmath} \times \vec{\imath})$.

6.For two non-zero vectors $\vec{a}$ *and* $\vec{b}$ write when $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ holds.

7. If$\vec{a}$, $\vec{b}$,$\vec{c}$, are unit vectors such that $\vec{a}$+$\vec{b}$+$\vec{c}$ = 0, find the value of $\vec{a}$.$\vec{b}$ +$\vec{b}$.$\vec{c}$ +$\vec{c}$.$\vec{a}$.

8.Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}= 3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b} = \hat{\imath}+2\hat{\jmath}-2\hat{k}$.

10. If $\vec{a},\vec{b},\vec{c}$ are the PVs of the vertices of A,B and C of a ΔABC respectively . Find an expression for the area of the ΔABC and hence deduce the condition that for the point A,B and C to be collinear.

11. If vector $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between then, show that $\sin\frac{\theta}{2}=\frac{1}{2}|\vec{a}-\vec{b}|$.

12 If vectors $a\hat{\imath}+\hat{\jmath}+\hat{k}$. $\hat{\imath}+b\hat{\jmath}+\hat{k}$ and $\hat{\imath}+\hat{\jmath}+c\hat{k}$ are coplanar show that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$.a,b.c $\neq 1$.

13. If with reference to the right handed system of mutually perpendicular unit vectors,$\hat{\imath},\hat{\jmath}$,and $\hat{k}$, $\vec{\alpha}=3\vec{\imath}-\vec{\jmath}$,$\vec{\beta}=2\vec{\imath}+\vec{\jmath}-\overrightarrow{3k}$, then express β in the form $\beta=\beta_1+\beta_2$.Where β_1 is parallel to $\propto$ and β_2 is perpendicular to $\propto$.

14. Let $\overrightarrow{a}=\vec{\imath}+4\vec{\jmath}-\overrightarrow{2k}$,$\vec{b}=3\vec{\imath}-2\vec{\jmath}+7\vec{k}$ and $\vec{c}=2\vec{\imath}-\vec{\jmath}+\overrightarrow{4k}$.Find a vector $\overrightarrow{d}$ which is perpendicular to both $\vec{a}$&$\vec{b}$ and $\overrightarrow{c}\cdot\vec{d}=15$.

CHAPTER – 11

THREE DIMENSIONAL GEOMETRY

CONCEPTS:-

Direction Cosines:

If a line makes an angle $\propto,\beta,\gamma$ with x, y and z axis respectively then the angles $\propto,\beta,\gamma$ are called direction angles of the line and the cosines of these angles are called the direction cosines of the line. The direction cosine of a line is usually denoted by l, m and n. i.e. l = Cos$\propto$, m = Cosβ and n = Cosγ . These are written as < l, m, n >.

Remarks: (1) For any line there are two sets of direction cosines.

(2) If l, m, n are direction cosines of a line then -l,-m,-n are also direction cosines of that line.

Direction Ratios:

Any three numbers a, b, c which are proportional to the direction cosines l, m, n of a line are called direction ratios (or direction numbers) of that line.

i.e. $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$.

Relation between Direction Cosines and Direction Ratios:

If l, m, n are direction cosines of a line then

$l^2 + m^2 + n^2 = 1$

$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$, $m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$ and $n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Straight Lines in space

Equation of a line passing through a given point A ($\vec{a}$) and parallel to a given direction$\vec{b}$ is given by:

$\vec{r} = \vec{a} + \lambda\vec{b}$,where λ is any scalar.

The Cartesian equation of a line passing through a given point A(x_1, y_1, z_1) and having direction ratios a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Equation of a line passing through two given points A ($\vec{a}$) and B ($\vec{b}$) is given by:

$\vec{r} = \vec{a} + \lambda(\overrightarrow{b} - \vec{a})$ and its Cartesian form is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

Angle between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$:

$$\text{Cos}\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

Cartesian form: $\text{Cos}\,\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}$

If lines are perpendicular then $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$, OR $a_1a_2 + b_1b_2 + c_1c_2 = 0$

#If lines are parallel then $\overrightarrow{b_1} \times \overrightarrow{b_2} = \vec{0}$ or $\overrightarrow{b_1} = k\overrightarrow{b_2}$ OR $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Shortest distance between two skew lines:

$$d = \left|\frac{(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2}).}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$$

Distance between two parallel lines is:

d = $\frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}|}{|\vec{b}|}$

EQUATION OF PLANES

Normal form:

Equation of a plane which is at distance d from the origin and perpendicular to a unit vector $\hat{n}$ is $\vec{r}.\hat{n}$ = d

Cartesian form: lx + my + nz = d where d.c's. of normal = l, m, n

Scalar product form:

Equation of a plane which passes through a given point A ($\vec{a}$) and it is perpendicular to $\vec{n}$ so, ($\vec{r}$-$\vec{a}$). $\vec{n}$= 0

Cartesian form: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where plane passes through a fixed point (x_1, y , z_1) and D.R of normal = a, b, c

General form:

ax + by + cz + d = 0 where D.R of normal = a, b , c

Intercept form:

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Equation of plane passing through three non collinear points in vector form is

$$(\vec{r}-\vec{a})\ .\ \{(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})\}=0$$

Equation of plane passing through three non collinear points in Cartesian form is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Angle between two planes : $\cos\theta = \frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{|n_1||n_2|}$

Angle between planes in Cartesian form

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

General equation of a plane passing through the line of intersection of two given planes

$\vec{r}.\overrightarrow{n_1}+d_1$ =0 and $\vec{r}.\overrightarrow{n_2}+d_2$ =0 is $\vec{r}.(\overrightarrow{n_1}+k\overrightarrow{n_2}) + (d_1+kd_2) = 0$

General equation of a plane passing through the line of intersection of two given planes

$A_1x+B_1y+C_1z+D_1 = 0$ and $A_2x+B_2y+C_2z+D_2 = 0$ is

$(A_1x+B_1y+C_1z+D_1)+ k (A_2x+B_2y+C_2z+D_2)= 0$

Distance of a plane $\vec{r}.\vec{n}$ = d from the point with p.v. $\vec{a}$ d= $\left|\frac{\vec{a}.\vec{n}-\mathrm{d}}{|\vec{n}|}\right|$

Distance of a plane ax +by +cz +d = 0 from the point (x_1, y_1, z_1) in Cartesian form:

$$\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

THREE LEVELS OF GRADED QUESTIONS:

LEVEL 1

1. Find the direction cosines of the line passing through the two points (-2,4,-5) and (1,2,3)
2. Find the direction cosines of X,Y,and Z-axes.
3. If $P(1,5,4)$ and $Q(4,1,-2)$ find the direction ratios of $\overrightarrow{PQ}$ and hence the direction cosines.
4. What is the cosine of the angle which the vector $\sqrt{2}$ i + j + k makes with y-axis.
5. Find the Vector and Cartesian equations for the line passing through the points (-1,0,2) and(3,4,6)
6. Find the angle between the pair of lines: $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$; $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$
7. Find the vector and the Cartesian equations of the line through the point (5,-2,3) and which is parallel to the vector 3i+2j-8k.
8. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.
9. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ *and* $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular , then find k.

10. Find the equation of the perpendicular drawn from the point (2,4,-1) to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

11. Find the coordinates of the image of the point P (1 , 3 , 4) in the plane 2x-y+z+3=0.

12. Prove that the image of the point (3, -2, 1) in the plane $3x - y + 4z = 2$ lies on the plane $x + y + z + 4 = 0$.

13. Find the length of the perpendicular from the point (2, -1, 5) to the line

$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.

14. Find the coordinates of the point where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meets the plane $x + y + 4z = 6$.

15. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane 10z+2y-11z = 3

16. Show that the lines $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ are intersecting. Hence find their point of intersection.

17. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3 , 0 , -4)

18. Find the shortest distance between the lines, whose equations are

$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{10-z}{-7}$ *and* $\frac{x-15}{3} = \frac{58-2y}{-16} = \frac{z-5}{-5}$.

19. Find the Co-ordinates of the point where the line through the point (5,1,6) and (3,4,1) crosses the YZ-plane.

LEVEL 2

1. Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured
parallel to the plane 4x+12y-3z+1=0.

2. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3 , 0 , -4). Also find the distance between these two lines. Also obtain the equation of the plane containing the line and the point (1, 2, 3).

3. Find the equation of the plane containing the lines $\vec{r} = \vec{\imath} + \vec{\jmath} + \lambda(\vec{\imath} + 2\vec{\jmath} - \vec{k})$and $\vec{r} = \vec{\imath} + \vec{\jmath} + \mu(-\vec{\imath} + \vec{\jmath} - 2\vec{k})$. Find the distance of the plane from origin and also from the point (1,1,1).

4. Find the equation of the perpendicular drawn from the point (2,4,-1) to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

5. Find the equation of the perpendicular drawn from the point P(2, 4, -1) to the line

$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also, write down the coordinates of the foot of the perpendicular from P to the line.

6. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.

7. Find the foot of the perpendicular from the point P(1, 2, 3)on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also obtain the equation of the plane containing the line and the point (1, 2, 3).

8. Find the coordinates of foot of perpendicular and the length of the perpendicular drawn from the point P (5,4,2) to the line $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + \hat{3}\jmath - \hat{k})$. Also find the image of P in this line.

9. Find the distance of the point (-2, 3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

10. Find the foot of the perpendicular from the point P(1, 2, 3)on the line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$$

Also obtain the equation of the plane containing the line and the point (1, 2, 3).

11. Find the equation of the plane containing the lines $\vec{r} = \hat{\imath} \vec{} + \hat{\jmath} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and

$\vec{r} = \hat{\imath} + \hat{\jmath} + \mu(-\hat{\imath} + \hat{\jmath} - \hat{k})$. Find the distance of the plane from origin and also from the point (1,1,1).

12. Find the distance of the point (3, 4, 5) from the plane $x + y = z = 2$ measured parallel to the line $2x = y = z$.

13. Find the distance of the point (-1 ,-5, -10) from the point of intersection of the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$.

14. Find the equation of the plane passing through the line of intersection of the planes

$\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$ *and* $\vec{r} \cdot (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + 4 = 0$and parallel to x –axis.

15. Find the equation of the plane passing through the intersection of the planes,

$2x + y - z = 3$, $5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

16. Find the distance between the point P(6, 5, 9) and the plane determined by thepoints A(3, -1, 2), B(5, 2, 4), and C(-1, -1, 6).

17. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

18. Show that the four points (0,-1,-1) (4,5,1) (3,9,4) and (-4,4 ,4) are coplanar. Also find the equation of the plane containing them

LEVEL 3

1. Find the equation of the plane through the intersection of the planes 2x +y-3z = 4 and

3x+4y+8z-1=0 and making equal intercepts on coordinate axes .
(Ans: 5x +5y+5z=3)

2. Find the equation of the plane containing the line $\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$ and perpendicular to the plane 2x-y+2z = 11.
(Ans: 2x-6y-5z = 3.)

3. Find the vector equation of the plane through the line of intersection of the planes

$\vec{r}$.(2$\hat{\imath}$ -3$\hat{\jmath}$+4$\hat{k}$) =1 and $\vec{r}$.($\hat{\imath}$ -$\hat{\jmath}$)+ 4 =0 and perpendicular to the plane $\vec{r}$.(2$\hat{\imath}$ -3$\hat{\jmath}$+4$\hat{k}$) =1

(*Ans*: $\vec{r}$.(-5$\hat{\imath}$ +2$\hat{\jmath}$+12$\hat{k}$) =47)

4. Find the equation of the plane perpendicular to the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ and passing through the point where the given line meets the plane x +y +4z =6 .

(Ans : 2x+3y+4z = 9.)

5. Find the equation of the plane passing through the line of intersection of the planes
$\vec{r}.(\hat{\imath} + 3\hat{\jmath}) - 6 = 0$ *and* $\vec{r}.(3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0$whose perpendicular distance from origin is unity.

6. Find the equation of the plane passing through the intersection of the planes
$2x + 3y - z + 1 = 0$*and* $x + y - 2z + 3 = 0$and perpendicular to the plane $3x - y - 2z - 4 = 0$ alos find inclination of this plane with XY-plane.

7. Find the vector and Cartesian equations of the plane passing through the intersection of the planes $\vec{r}.\ (\hat{\imath} + \hat{\jmath} + \hat{k})$ = 6 and $\vec{r}.\ (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ = -5 and the point (1,1,1)

8. Let P(3,2,6) be a point in the space and Q be a point on the line $\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$ then find the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x – 4y + 3z = 1.

ONE MARK QUESTIONS

1. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ represent sides of a triangle taken in order then value of $\vec{a} + \vec{b} + \vec{c}$.
2. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ represent vertices of a triangle ABC then find the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.
3. Write a unit vector making equal acute angles with the coordinate axes.
4. *If* $sin^2\alpha + sin^2\beta + sin^2\gamma = k$ where α, β *and*γ *make* angles with x y and z axis then find k.
5. *Find the dcs of the* $\vec{r} = 6\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.
6. If $\vec{a} = \hat{\imath} + \hat{\jmath}, \vec{b} = \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} + \hat{k}$,write a unit vector parallel to $\vec{a} + \vec{b} - 2\vec{c}$.

7. If $\vec{a}= 3\hat{\imath} - \hat{\jmath} - 4\hat{k}$, $\vec{b}=-2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{c}= \hat{\imath} + 2\hat{\jmath} - \hat{k}$ find magnitude of $3\,\vec{a} - 2\vec{b} + \overrightarrow{4c}$

8. If a line makes angles 90^0 , 135^0 and 45^0 with X, Y and Z axis respectively then find its D.Cs

9. If a line has direction ratios -18, 12 and -4 then what is its D.Cs.

10. Find the length of the x-intercept cut off by the plane 2x + y – z = 5.

11. Find the angle between the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath}$-$2\hat{\jmath} + \hat{k}$

12. Find the projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}.\ \vec{b}$ =8 and $\vec{b}$ =$2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.

13. Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z+5}{6}$.

14. The Cartesian equation of a line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$. Write the direction ratios.

15. Find the distance between the planes 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12.

16. Find the angle between the planes whose vector equations are $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.

17. Find the Cartesian equation of the plane $\vec{r}.\big((s-2t)\hat{\imath} + (3-t)\hat{\jmath} + (2s+t)\hat{k}\big) = 15$

18. Find the perpendicular distance of the plane x + y + z =1 from origin.

19. Find the equation of a line parallel to X-axis and passing through the origin.

20. The plane 2x-3y+6z =11 makes an angle $\text{Sin}^{-1}\alpha$ with the X-axis. Find α.

Answers:

1. 0 2. 0 3. $\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$ 4. 2 5. $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ 6. $\frac{1}{\sqrt{6}}(-\hat{\imath} + 2\hat{\jmath} - \hat{k})$ 7. $\sqrt{398}$

8. 0, $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ 9. -9/11, 6/11 and -2/11 10. 5/2 11. $\text{Cos}^{-1}5/7$

12. 8/7 13. $\vec{r}.=5\hat{\imath}-4\hat{\jmath}-5\hat{k}+\lambda(3\hat{\imath}+7\hat{\jmath}+6\hat{k})$ 14)1,-7,2

15) $2/\sqrt{29}$, 16. $\text{Cos}^{-1}15/\sqrt{731}$,17. $(s-2t)x+(3-t)y+(2s+t)z=15$

18) $\frac{1}{\sqrt{3}}$ 19) $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$. 20) 2/7

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CHAPTER – 12

LINEAR PROGRAMMING PROBLEM:

LEARNING OBJECTIVES/OUTCOMES:

- To understand the nature of Linear Programming Problem.
- To know about the Decision Variables.
- To acquire the concept (meaning) of Objective Function.
- To know about the Non-Negative Constraints.
- To construct the Structural Constraints (Inequalities).
- To understand the concept of Feasible Solution.
- To find the Optimum Solution from the Feasible Solution.
- To develop skills to find the Optimum Solution for the Unbounded Feasible Region.
- To be able to solve all practical problems of LPP.
- Linear Programming is a mathematical technique for determining the optimum allocation of resources and obtaining a particular objective when there are alternative uses of the resources, money, manpower, material, machine and other facilities.

TERMINOLOGY USED IN LINEAR PROGRAMMING PROBLEM

Components of LP Problem: Every LPP is composed of

1.Decision Variable, 2. Objective Function, 3. Constraints.

4.Feasibile region. 5. Corner points 6. Optimization values

QUESTIONS:

1. Graphically solve the system of inequations
   $x-y \leq 2$, $x+y \leq 4$, $x \geq 0$, $y \geq 0$

2. Minimise $Z = 2x + 3y$ subject to constraints : $x \geq 0$, $y \geq 0$, $1 \leq x+2y \leq 10$

3. A producer has 20 and 10 units of labor and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of goods X, 2 units of capital and 1 unit of labor is required. To produce one unit of goods Y, 3 units of labor and one unit of capital is required. If X and Y are priced at Rs.80 and Rs.100 per unit respectively, how should the producer use his resources to maximize the total revenue?. Solve the problem graphically.

4. A small firm manufactures items A and B. The total number of items A and B that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs.300 and one unit of item B be Rs.160, how many of each type of item be produced to maximize the profit? Solve the problem graphically.

5. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88gms of protein and at least 36 milligrams of iron. Knowing that bran contains 80gms of protein and 40 milligrams of iron per kilogram, and that rice contains 100gms of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs.5 per kilogram and rice costs Rs.4 per kilogram.

6. A firm manufactures two types of products A and B and sells them at a profit of Rs.5 per unit of type A and Rs.3 per unit of type B. Each product is produced on two machines M_1 and M_2, whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

7. A factory owner wants to purchase two types of machines, A and B , for his factory. The machine A requires an area of 1000 m^2 and 12 skilled men for running it and its daily output is 50 units, whereas the machine

B requires 1200m^2 and 8 skilled men and its daily output is 40 units. If an area of 7600m^2 and 72 skilled man be available to operate the machine, how many machines of each type should be bought to maximize the daily output.

QUESTIONS (1-marks)

Fill in the blanks

1. Feasible solution satisfies _____

A. Only constraints B. only non-negative restriction

C. [a] and [b] both D. None of Them

2 . Minimize Z = _____

A. - maximize(Z) B. maximize(-Z) C. -maximize(-Z) D. none of these

3. In graphical method the restriction on number of constraint is: _

A. 2 B. 3 C. not more than 3 D. none of the above

4. In graphical representation the bounded region is known as _____ region.

A. Solution B. feasible solution C. basic solution D. optimal

5. Graphical optimal value for Z can be obtained from

A. Corner points of feasible region

B. corner points of the solution region

C. Both a and c

D. none of the above

6. In LPP the condition to be satisfied is

A. Constraints have to be linear

B. Objective function have to be linear

C. both [a]and [b]

D. none of the above

7. The solution to LPP give below is,

Max Z=3 x +14 y, subject to , where x - y ≥1 ; −x + y≥ 2; x,y ≥0

A. Unbounded solution B. Max Z = 14 C. Max Z = 3 D. Infeasible solution

8. The solution to LPP give below is _____

Max Z=30x-15y; subject to x-y ≤1 ; -2x +2y ≤ 2 ; x,y≥0

A. Unbounded solution C. Max Z = 15 B. Max Z = 30 D. Infeasible solution

9. The linear function of variables which is to be maximized or minimized is called _____

A. constraints C. objective function B. basic requirements

D. none of them

10.A feasible solution of LPP

A. Must satisfy all the constraints simultaneously

B. Need not satisfy all the constraints, only some of them

C. Must be a corner point of the feasible region

D. all of the above

11. The objective function for a L.P model is $z= 3x+2y$; if $x=20$ and $y=30$, what is the value of the objective function?

A. 0 B. 50 C. 60 D. 120

12. Alternative solution exist in a linear programming problem when

A. one of the constraint is redundant

B. objective function is parallel to one of the constraints

C. two constraints are parallel D. all of the above

13. The linear function of the variables which is to be maximize or minimize is called ---------

A. Constraints B. Objective function C. Decision variable D. None of the above

14. If the value of the objective function z can be increased or decreased indefinitely, such solution is called _____

A. Bounded solution B. Unbounded solution C. Solution D. None of the above

Answer Key:

1.C . 2D , 3.D ,4.B ,5.A ,6.C ,7.A ,8.D 9.C, 10.A, 11.D, 12.B, 13.B 14.B

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CHAPTER -13
PROBABILITY

BASIC CONCEPTS AND KEY POINTS

1. Conditional Probability: If A and B are two events associated with the same random experiments, then the probability of occurrence of event A when the event B has already occurred is called conditional probability of A when B is given. It is represented by P(A/B) and is given by

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

2. Multiplication Theorem on Probability: Let A and B be two events associated with sample space S. Then $(A \cap B)$ denotes the event that both A and b have occurred. $(A \cap B)$ denotes the simultaneous occurrence of the events A and B.

$P(A \cap B)$ =P(A) P(B/A) = P(B) P(A/B)

3. Independent Events: Two events A and B such that the probability of occurrence of one of them is not affected by the occurrence of the other. If two events are independent then the probability that both will occur is equal to the product of their individual probabilities.

$P(A \cap B)$ =P(A). P(B)

This theorem can be extended for three or more independent events

i.e. $P(A \cap B \cap C)$ =P(A). P(B) .P(C)

4. Theorem of total probability: Let $\{E_1, E_2, \ldots\ldots\ldots, E_n\}$ be a partition of the sample space S, and suppose that each of the events $E_1, E_2, \ldots\ldots\ldots, E_n$ has a non-zero probability of occurrence . Let A be any event associated with S then

$P(A) = P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) + \ldots\ldots\ldots\ldots\ldots\ldots + P(E_n)\, P(A/E_n)$

$= \sum_{j=1}^{n} P(E_j)\, P(A/E_j)$

5. Baye's Theorem: If $E_1, E_2, \ldots\ldots, E_n$ are non-empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots\ldots, E_n$ are

pairwise disjoint and $E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$ and A is any event of nonzero probability, then

$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_i)P(A/E_i)}$ for any i=1,2,3,……n

Mean of a Discrete Random Variable

The mean of the discrete random variable X is also called the **expected value** of X is denoted by E(X). Use the following formula to compute the mean of a discrete random variable.

$E(X) = \mu_x = \Sigma [x_i * P(x_i)]$ where x_i is the value of the random variable for outcome i, μ_x is the mean of random variable X, and $P(x_i)$ is the probability that the random variable will be outcome i.

Variance of X = $V(X) = E(X^2) - [E(X)]^2$, **Standard deviation** = $\sqrt{Variance}$

THREE LEVELS OF GRADED QUESTIONS:

LEVEL - 1

1. If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive events
2. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced.
3. If P(A) = 6/11 , P(B) = 5/11 and P(A ∪ B)= 7 /11,
   find (i) P (A∩B) (ii) P (A|B) (iii) P(B|A)
4. Two cards are drawn at random and without replacement from a pack of 52
   playing cards. Find the probability that both the cards are black
5. A die is thrown 6 times. If 'getting an odd number' is a success, what is the
   probability of (i) 5 successes? (ii) at least 5 successes? (iii) at most 5 successes?

6. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?
7. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red
   and 6 black balls. One ball is drawn at random from one of the bags and it is found tobe red. Find the probability that it was drawn from Bag II.

8. A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what the probability that none is red is.
9. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag. Find the probability that the ball drawn is blue in colour.
10. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of spades

Level – 2

1. A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find the probability that none is red.
2. The probability of A hitting a target is 3/7 and that of B hitting is 1/3. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.
3. An insurance company insured 2000 scooter and 3000 motorcycles. The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02. An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle.

4. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin?
5. A company has two plants to manufacture bicycles. The first plant manufactures 60 % of the bicycles and the second plant 40 %. Out of that 80 % of the bicycles are rated of standard quality at the first plant and 90 % of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

6. A family has 2 children. Find the probability that both are boys, if it is known that (i) at least one of the children is a boy. (ii) the elder child is a boy**.**
7. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is

drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

8. 4 defective apples are accidentally mixed with 16 good ones. Three apples are drawn at random from the mixed lot. Find the probability distribution of the number of defective apples
10. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

LEVEL- 3

1. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON?

2. A test detection of a particular disease is not fool proof. The test will correctly detect the disease 90 % of the time, but will incorrectly detect the disease 1 % of the time. For a large population of which an estimated 0.2 % has the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease

3. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

4. In a school there are 1000 students , out of which 430 are girls. It is known that out of 430 , 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl?

PROBABILITY (01 /02 MARK QUESTIONS)

Q.1) Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{8}$, then the probability that the problem is solved correctly by at least one of them is

(a) $\frac{235}{256}$ (b)$\frac{21}{256}$ (c) $\frac{3}{256}$ (d) $\frac{253}{256}$ Ans: (a)

Q.2) Four dice are thrown simultaneously. Find the probability that all of them show the same face.

(a) $\frac{4}{216}$ (b)$\frac{1}{216}$ (c) $\frac{3}{216}$(d) $\frac{1}{36}$ Ans: (b)

Q .3) A bag contains 12 white and 18 black balls. Two balls are drawn in succession without replacement.What is the probability that first is white and second is black?

Q.4 If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive event. Ans. 0

Q.5 A die is thrown 6 times. If getting an odd number is a success. What is the probability of 5 successes? Ans. 3/32

Q.6 A couple has 2children. Find the probability that both are boys, if it is known that one of them is a boy . Ans. 1/2

Q.7 If P (AIB) = x , P(B) =0.5 and P(A∩ B) = 0.32 .Find x
Ans. 16/25

Q.8 Find k if X is a Random variable having distributions$P(X) =$

$$\begin{cases} k\,, if\ X = 0 \\ 2k\,, if\ x = 1 \\ 3k\,, if\ X = 2 \\ 0, ohterwise \end{cases}$$

Ans. 1/6

Q.9 X is random variable and has following probability distributions

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find P(X<3) Ans. 3/10

CLASS: XII, MATHEMATICS (Code-041) BLUE PRINT

UNIT	NAME OF THE CHAPTER/TOPIC	VSA (1 Mark)	VSA (2 Marks)	LA-I (4 Marks)	LA-II (6 Marks)	TOTAL
I.Relations and Functions	1. Relations and Functions	1(1)	***	***	1 (6)	**08**
	2. Inverse Trigonometric Functions	1(1)	***	***	***	
II. Algebra	3. Matrices	1 (1)	2(4)	***	***	**10**
	4. Determinants	1 (1)	***	1(4)	***	
III. Calculus	5. Continuity and Differentiability	2(2)	***	1(4)	***	**35**
	6. Applications of Derivatives	1(1)	1(2)	1(4)	*	
	7. Integrals	2(2)	1(2)	1(4)	1(6)	
	8. Applications of Integrals	***	***	***	***	
	9. Differential Equations	4(4)	***	1(4)	***	
IV. Vectors and 3-D	10. Vector Algebra	4(4)	1(2)	***	***	**14**
	11. Three- Dimensional Geometry	2(2)	***	***	1 (6)	
V. Linear Programming	12. Linear Programming	1(1)	***	1(4)	***	**05**
VI. Probability	13. Probability	***	1(2)	***	1(6)	**08**
	TOTAL	**20(20)**	**6(12)**	**6(24)**	**4(24)**	**36(80)**

Kendriya Vidyalaya Sangathan, Kolkata Region

Sample Question Paper-2019-20 Paper-1

Mathematics (041)

Class –XII

Time allowed: 3 hours Maximum Marks: 80

General Instructions:

(i) All questions are compulsory.

(ii) This question paper contains 36 questions.

(iii) Question 1- 20 in Section A are very short-answer type questions carrying 1 mark each.

(iv) Question 21-26 in Section B are short-answer type questions carrying 2 marks each.

(v) Question 27-32 in Section C are long-answer-I type questions carrying 4 marks each.

(vi) Question 33-36 in Section D are long-answer-II type questions carrying 6 marks each.

Section : A

1. If R = {(a, a^3) : a is a prime number less than 5} be a relation. Find the range of R.
2. Find the slope of the tangent to the curve y = 3 x^2–4 x at point whose x-coordinate is 2
3. A is a square matrix of order 3 and $|A|$ = 6. Write the value of $|adj.A|$.
4. Write a vector of magnitude 8 units in the direction of vector $-2\hat{i}+\hat{j}+2\hat{k}$.
5. A line passes through the points A (0, 3, 2) and B (4, – 1, 6). Then direction ratios of AB are _____.
6. If $\vec{x}$ is a unit vector suct that $\vec{x} \times \hat{j} = \hat{k}$, find $\vec{x} \cdot \hat{j}$.
7. Write the position vector of the midpoint of the vector joining the points P (2 ,3 ,4) and Q (4 ,1 ,-2).

8. Let $\vec{a}$ & $\vec{b}$ be two vectors such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}$ x $\vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ & $\vec{b}$.

9. If f(x) = $f(x) = \int_0^x t \sin t \, dt$, *then write the value of* $f'(x)$.

10. Write the sum of the order and degree of the differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$.

11. The Cartesian equation of a line is 2x-3 = 3y+1 = 5-6z. Write the direction ratios.

12. If A = $\begin{vmatrix} 2 & 5 \\ 3 & -2 \end{vmatrix}$ then find $|A^{-1}|$.

13. Find the value of cot ($\sec^{-1}x + \sec^{-1}\frac{1}{x}$).

14. Find the differential equation of the family of lines passing through origin.

15. The function f (x)= $\frac{x^2-49}{x-7}$, x≠ 7 ,x∈ R is a continuous function. State true or false.

16. What is meant by constrains in a linear programming?

17. Find the value of c in Rolle's theorem when f(x)= $2x^3 - 5x^2 - 4x + 3$, x ∈ $[\frac{1}{3}, 3]$.

18. Evaluate $\int e^x(\tan x + 1)\sec x \, dx$.

19. Write the differential equation representing the family of the curve y = m x where m is an arbitrary constant.

20. y = a cos x + b sin x is a solution of differential equation

a) y"+ y = 0 b) y'+ y=0

c) y"+ 3y =0 d) y"+ aby=0

SECTION :B

21. Show that all the diagonal elements of a skew symmetric matrix are zero.

22. Find matrices X and Y if X+Y=$\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$ and X-Y= $\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$.

23. Test the continuity of the function at the origin

f (x)=$\begin{cases} \frac{|x|}{x}\ ; & x \neq 0 \\ 1 & x = 0 \end{cases}$

24. Integrate $\int sin^2 x e^{logcosx} dx$.

OR

$\int \frac{1}{1+tanx} dx$

25. Find λ , when projection of vector $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + \hat{6J} + 3\hat{k}$ is 4 units.

26. Prove that if E and F are independent events, the events E and F' are also independent events.

Section : C

27. If α, β,+ are the roots of the equation $x^2(px+q)=r(x+1)$, prove that $\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix}$=0.

28. If the function$f(x) = \begin{cases} 3ax + b, if\ x > 1 \\ 11, if\ x = 1 \\ 5ax - 2b, if\ x < 1 \end{cases}$ is continuous at $x = 1$.

Then find the value of $a\ and\ b$.

29. Evaluate $\int \frac{3x+1}{\sqrt{5-2x-x^2}}\ dx$.

30. Using differential, find approximate value of $\sqrt{4.5}$.

31. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$. Given that y=1 when x = 0.

32. Solve the following LPP graphically.

Minimise $z = 5x + 10y$, subject to the constraints

$x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$ and $x \geq 0$, $y \geq 0$.

Section : D

33. Show that the relation R defined by

$R = \{((a, b), (c, d)\}: ad(b + c) = bc(a + d)\}$ defined in N x N is an equivalence relation where N is the set of natural numbers.

OR

Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with $f^{-1}(y) = \sqrt{y - 4}$ where R_+ is the set of all non-negative real numbers.

34. Find image of the point (1, 3, 4) in the plane $\vec{r} . (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

35. Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

OR

Using integration, find the area of the region bounded by the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

36. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also, find mean and variance of the distribution.

	MODEL PAPER-2 SUBJECT: MATHEMATICS Class- XII TIME : 3 HRS. Max. Marks :80	
	SECTION A	
1	Find the principal value of $\cos^{-1}(\cos\frac{7\pi}{6})$	1
2	Find the value of $\tan^{-1}\sqrt{3}$ - $\sec^{-1}(-2)$	1
3	If A is a square matrix of order 3 such that $\|Adj(A)\| = 9$, find $\|A^T\|$.	1
4	If A is a matrix of order 3 and det(A) = 4 , find det(3A).	1
5	If A is a matrix of order m×3 and B is a matrix of order $3\times p$ then what will be the order of matrix AB ?	1
6	Find values of x, if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$	1
7	For what value of "k" the function f (x) = $\begin{cases} \frac{sin5x}{3x} + cosx & if\ x \neq 0 \\ k, if\ x = 0 \end{cases}$, is continuous at x = 0.	1
8	Find the slope of tangent to the curve $y = 3x^2 - 6$ at the point on it whose x- coordinate is 2.	1
9	Find the maximum profit that a company can make if the profit function is given by $p(x)= 41-72x-18x^2$.	1
10	Integrate $e^x(\frac{1}{x} - \frac{1}{x^2})$.	1
11	Write the anti-derivative of $\frac{x+cos6x}{3x^2+sin6x}$	1
12	Write a vector in the direction of the vector $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ that has magnitude 9 units.	1

,13	Write the value of p for which the vectors $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$ are parallel.	1
14	The corner points of the feasible region determined by following system of inequalities: $2x+y\leq 10$, $x+3y\leq 15$, $x, y\geq 0$ are (0,0),(5,0),(3,4) and (0,5). Let $Z=px + qy$, where p, $q>0$. Find condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5).	1
15	Find the projection of the vector $7\hat{\imath} + \hat{\jmath} - 4\hat{k}$ on the vector $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$	1
16	If a line makes angle 90^0 , 60^0 and θ with x , y and z-axis respectively where θ is acute, then find θ.	1
17	If P(E)=$\frac{1}{4}$ and P(F)=$\frac{1}{2}$ and P(E∩F)= $\frac{1}{8}$, then find P(E/F).	1
18	Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.	1
19	If E and F are two events such that P(E)=$\frac{1}{4}$ and P(F)=$\frac{1}{2}$ and P(E∩F)= $\frac{1}{8}$, find P(not E and not F).	1
20	Write the integrating factor of differential equation $x\frac{dy}{dx} + y = x^2$	1
	(SECTION B	
21	The volume of a cube is increasing at the rate of 9 cm^3/sec.How fast is the surface area increasing when the length of an edge is 10 cm.	2
22	$\left\|\vec{a} + \vec{b}\right\| = 60$, $\left\|\vec{a} - \vec{b}\right\| = 40$ and $\|\vec{a}\| = 22$, then find $\left\|\vec{b}\right\|$.	2
23	Write in simplest form $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}, x \neq 0$	2

24	An Apache helicopter of enemy is flying along the curve given by $y=x^2+7$. A soldier, placed at (3,7) wants to shoot down the helicopter when it is nearest to him. Find the nearest distance. OR Find the absolute maximum and minimum values of $f(x)=2x^3-15x^2+36x+1$ on [1,5].	2
25	If the lines $\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{z-1}{-7}$ are perpendicular to each other, then find the value of p.	2
26	Find the mean number of heads in two tosses of a coin.	2
	SECTION C	
27	If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $-1 < x < 1$, $x \neq 0$. Find $\frac{dy}{dx}$. OR If $(cosx)^y = (cosy)^x$, then find $\frac{dy}{dx}$.	4
28	Solve the differential equation $(\cot^{-1}y + x)dy = (1 + y^2)dx$.	4
29	Solve the following linear programming problem graphically: Maximise Z = 4x + y Subject to the constraints $x + y \leq 50$, $3x = y \leq 90$, $x \geq 10$, $x,y \geq 0$.	4
30	Show that the relation R in the set A = {1,2,3,4,5} given by R = {(a,b) : $\lvert a-b \rvert$ is even} is an equivalence relation..Show that all the elements of {1,3,5] are related to each other and all the elments of {2,4} are related to each other. But no element of {1,3,5] is relaed to any element of {2,4}. OR Consider $f: R_+ \rightarrow [4,\infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible and find f^{-1} , where R_+ is the set of all non-negative real numbers.	4

31	Evaluate $\int \frac{x^2+1}{(x-1)^2\,(x+3)}$dx.	
32	A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at randomfrom one of the bags and is found to be red . Find the probability that the ball was drawn from bag B.	4
	SECTION D	
33	Evaluate $\int_1^3 (2x^2 + 5x)$dx as the limit of sum OR Evaluate $\int_0^{\pi} \frac{xsinx}{1+sinx}$	6
34	Solve the following equations by using matrix method x+y+z=12, 3y+3z+2x=33, x+z=2y OR Using properties of determinants, prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$	6
35	If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of 'k'and hence find the equation of the plane containing them.	6
36	Sketch the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2=6y$.Also find the area of the region using integration. OR Find the area of the region bounded by the parabola $y^2= 2x$ and the straight line $x - y = 4$.	6

MODEL SAMPLE PAPER - 3

SUB:- MATHEMATICS CLASS – XII

SECTION- A [1 MARK EACH]

1. In the set A = {1,2,3} given by R = {(1,2),(2,3)}. Is it transitive?
2. Find the value of $\tan^{-1}\sqrt{3} + \sec^{-1}(-2)$
3. Write the value of $\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$
4. If $\begin{bmatrix} x & 2 \\ 18 & x \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 18 & 6 \end{bmatrix}$, then value of x is

   (a) 6 (b) 0, (c) ± 6 (d) $\frac{1}{6}$
5. If $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and A_{ij} is Cofactors of element a_{ij} , then value of $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ is

   (a) Δ (b) 0 (c) 1 (d) cannot be determined
6. If A is a square matrix of order 3 such that $|adjA| = 64$, find $|A'|$.
7. If y = sin 5x, find Y_2.
8. Find the derivative of $e^{\log x}$ with respect to x.
9. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 4 cm.
10. Find the slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at t =2.
11. Find the value of the integral: $\int_{-1}^{1} x^{17} \cos^4 x \, dx$.
12. Write the order and degree of $\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}$
13. A differential equation $\frac{dy}{dx} = F(x,y)$ is homogeneous if

    (a) $F(\lambda x, \lambda y) = \lambda^2 F(x,y)$ (b) $F(\lambda x, \lambda y) = \lambda F(x,y)$

    (c) $F(\lambda x, \lambda y) = \lambda^0 F(x,y)$ (d) $F(\lambda x, \lambda y) = \lambda^{-1} F(x,y)$
14. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ *and* $\vec{a}.\vec{b} = 3$, *find the angle between* $\vec{a}$ *and* $\vec{b}$.

15. Write the value of $(\hat{i}\times\hat{j}).\hat{k}+\hat{i}.\ \hat{j}$.

16. Write the direction cosines of a line equally inclined to the three co-ordinate axes.

17. Write the vector equation of a line given by $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$.

18. If P(A)=3/5 and P(B)=1/5, find P (A∩ B) if A and B are independent events.

19. Let E and F be events with P(E) =3/5, P(F) =3/10 and P($E\cap F$) = 1/5 . Find the value of P(F/E).

20. Of all the points of the feasible region, for maximum or minimum of objective function, the point lies

(a) Inside the feasible region

(b) At the boundary line of the feasible region

(c) Vertex point of the boundary of the feasible region

(d) None of these

SECTION- B [2 MARKS EACH]

21. Prove that : $2\tan^{-1}\left(\frac{2}{3}\right)=\cos^{-1}\frac{5}{13}$

22. If the side of cube be increased by 0.1%,find the corresponding increase in the vol. of cube.

23. Evalute:- $\int(\frac{x-1}{x^2})\ e^x\ dx$ =f(x) e^x +c then the value of f(x) .

24. Write a vector of magnitude 9 units in the direction of vector $-2\hat{i}+\hat{j}+2\hat{k}$.

25. Find the angle θ, between the line $\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{4}$ and the plane 2x - 2y + z - 5 = 0.

26. A random variable X has the following probabilities distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Determine the value of k.

SECTION- C [4 MARKS EACH]

27. Consider f : R+→(-5, ∞) given by f(x) = 9 x^2 + 6x -5. Show that f is invertible with

$$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$$

28. Determine the value of 'a' so that the following function is continuous.

$$f(x) = \begin{cases} a\,x^2 + b, & x\rangle 2 \\ 2, & x = 2 \\ 2a\,x - b, & x\langle 2 \end{cases}$$

29. Evaluate: $\int \frac{dx}{x(x^n+1)}$

OR

Evaluate: $\int \frac{x+1}{(x+3)^3} e^x\, dx$

30. Find the equation of the plane through $2\hat{i} + \hat{j} - \hat{k}$ and passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $\vec{r} \cdot (\hat{j} + 2\hat{k}) = 0$.

31. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.

[OR]

Find the vector equation of the plane passing through (1, 2, 3) and perpendicular to the plane $\vec{r} \cdot \left(\hat{i} + 2\hat{j} - 5\hat{k}\right) + 9 = 0$

32. In a bolt factory, three machines A, B, C manufacture 25%, 35% and 40% of the total production respectively. Of their respective output, 5%, 4% and 2% are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine C.

SECTION- D [6 MARKS EACH]

33. Obtain the inverse of the following matrix using elementary operations.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

OR

Solve the system of equations by matrix method:

$$3x + 4y + 2z = 8$$
$$2y - 3z = 3$$
$$x - 2y + 6z = -2$$

34. Show that the semi vertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

35. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

OR

Calculate the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $\frac{x}{4} + \frac{y}{3} = 1$

36. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ *and* $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Also find the equation of the plane containing them.

MODEL PAPER – 4

CLASS – XII SUBJECT-MATHEMATICS

Time allowed: 3 hours **Maximum Marks : 80**

General Instructions: same as paper- 1

SECTION - A

1. If the function f : R → R given by f(x) = x^2 + 4 and g: R → R given by g(x) = $\frac{x}{x-1}$, x ≠ 1,find fog.

2. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then find x.

3. Use elementary column operation $C_2 \rightarrow C_2 - 2C_1$ in the following matrix equation:

4. $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$.

5. If A is an invertible matrix of order 3 and$|A|$ = 5, then find $|adjA|$.

6. If y = e^{logx^5}, find $\frac{dy}{dx}$.

7. Find the value of for which the function f(x)=x^2−2ax + 6, x > 0 is strictly increasing.

8. Find the point on the curve y=x^2−2x+3 where the tangent is parallel to x-axis.

9. Evaluate : $\int (\frac{\cos 2x + 2\sin^2 x)}{\cos^2 x} dx$

10 Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx$

11. Find the co-ordinates of the point on the curie y^2 = 3 – 4x, where tangent is parallel to the line 2x + y –2 = 0

12.Find $\int e^x 2^x dx$

13. Write the sum of the order and degree of the differential equation

$\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0$

14.

Verify that $ax^2 + by^2 = 1$ is a solution of the differential equation $x(yy_1 + y_1^2) = yy_1$.

15. Find the projection (vector) of $2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

16. If $\vec{a}$ is a unit vector and$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8$, then find$|\vec{x}|$.

17. What is the angle between lines 2x=3y=-z and 6x =-y=-4z.

18. What is the equation of the plane which cuts off equal intercepts of uinit length on coordinates axis.

19. What are the conditions for unbounded solution?

20. Prove that if E and F are independent events, then the events E and F' are also independent.

SECTION - B

21. Prove that if ½ ≤ x ≤ 1 , then $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$

22. Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

23. Find $\int \frac{\sec x}{1+cosec\, x}$ dx

24. Solve the diff. eq. $\dfrac{dy}{dx} = e^{x+y} + x^2 e^y$

25. Find the relationship between 'a' and 'b' so that the function 'f' defined by :

$$f(x) = \begin{cases} ax + 1, & x \le 3 \\ bx + 3, & x > 3 \end{cases} is continuous at x = 3.$$

OR

Find all the point of discontinuity of the function f defined by

$$f(x) = \begin{cases} x + 2, & if\ x \le 1 \\ x - 2, & if\ 1 < x, 2 \\ 0\ , & if\ x \ge 2 \end{cases}$$

26. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III.

SECTION - C

27. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution.

28. A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and Rs1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and Rs 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of Rs 9 per unit and the second product described has a market value (sale price) of Rs 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

29. Find the particular solution of the differential equation: $y\, e^{y}\, dx = (y^3 + 2xe^{y})\, dy$, $y(0) = 1$

OR

Show that $(x - y)\, dy = (x + 2y)$ is a homogenous differential equation. Also, find the general solution of the given differential equation.

30. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

31. Let f: N $\rightarrow$R be a function defined as f(x) = $4x^2$ + 12x +15. Show that f: N s is invertible, where S is the range of f. hence. Find inverse of f.

32. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

Section D

33. *Find* $\int (x+3)\sqrt{3-4x-x^2}\ dx$.

OR

Evaluate the following integral as limit of sums $\int_1^4 (x^2 - x)dx$

34. Find the area of the region in the first quadrant enclosed by the x-axis,the line y= x and the circle $x^2+y^2=32$.

35.

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: $3x+4y+7z=14, 2x-y+3z=4, x+2y-3z=0$

36.

Find the distance of point $-2\hat{i}+3\hat{j}-4\hat{k}$ from the line

$\vec{r} = \hat{i}+2\hat{j}-\hat{k}+\lambda(\hat{i}+3\hat{j}-9\hat{k})$

measured parallel to the plane: $x - y + 2z - 3 = 0$.

SAMPLE PAPER-05

CLASS XII MATHEMATICS

TIME : 3 HRS. Max. Marks :80

SECTION A	
1	Find *fog(7),* if *f(x)= x+7 & g(x)=x-7.*
2	If A is a square matrix of order 3 such that $\|adjA\| = 9$, find$\|A^T\|$.
3	If the side of cube be increased by 0.1%,find the corresponding increase in the vol. of cube.
4	Find the value of λ if the sum of two vectors $\hat{i} + \hat{j} - \lambda\hat{k}$ & $\hat{i} - \hat{j}$ is perpendicular to $2\hat{i} - \hat{j} + \hat{k}$.
5	Is there any feasible solution for the following constraints: 2x+y ≥ 8,4x+2y≤ 32?
6	Find the rate of change of the area of a circle with respect to its radius 'r' when r = 4 cm.
7	Write the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$
8	What is the co-efficient of the highest order differential in the differential equation of $y^2 = 4ax$?
9	If y = e^x + e^{-x}, then find Y_2 .
10	Write the adjoint of the matrix $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$
11	Evaluate : $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\,dx$
12	Find the slope of the tangent to the curve x = t^2 + 3t – 8, y = $2t^2$ – 2t -5 at t =2.
13	Find the value of the integral: $\int_{-1}^{1} x^{17}\cos^4 x\,dx$.
14	If $\|\vec{a}\| = \sqrt{3}, \|\vec{b}\| = 2$ *and* $\vec{a}.\vec{b} = 3$, *find the angle between* $\vec{a}$ *and* $\vec{b}$.

15	If the radius of a circle increases from 5 cm to 5.1 cm, find the increase in area.
16	If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix}$ = 3, find the value of x
17	Give an example of a function which is continuous at $x = 1$, but not differentiable at $x = 1$.
18	What should be the order of the differential equation whose general solution is : Y= a sin px + b cos px
19	Write the value of $(\hat{i} \times \hat{j}).\hat{k} + \hat{i}.\ \hat{j}.$
20	What is the maximum no of 3 X 3 matrices with each element either 0 or 1?
SECTION B	
21	Prove that: $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$
22	Let $f(x) = \frac{tan(\frac{\pi}{4} - x)}{cot\ 2x}, x \neq \frac{\pi}{4}$. Find the value which should be assigned to f(x) at $x = \frac{\pi}{4}$, so that it is continuous
23	If $y = x^{\sin x - \cos x}$ *find* $\frac{dy}{dx}$.
24	Evaluate: $\int \frac{2x+1}{x^2-1} dx$
25	Find the angles which the line $\frac{x}{1} = \frac{y-1}{-1} = \frac{z+1}{1}$ makes with axes.
26	A and B throw a pair of die turn by turn.The first to throw nine is awarded a prize.If A starts the game , show that the probability of A getting the prize is 9/17
SECTION C	
27	Let f : W W be a function as

	$f(n)=\begin{cases} n-1, & if\ n\ is\ odd \\ n+1, & if\ n\ is\ even \end{cases}$ Show that f is invertible and find the inverse of f. here, W is the set of all whole numbers.
28	Let $f(x)=\begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if } x<0 \\ a, & \text{if } x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{if } x>0 \end{cases}$ Determine the value of a so that f(x) is continuous. OR If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.
29	Find the shortest distance between the lines whose vector equations are $\vec{r}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})+\lambda(\hat{\imath}-3\hat{\jmath}-2\hat{k})$ and $\vec{r}=(4\hat{\imath}+5\hat{\jmath}+6\hat{k})+\mu(2\hat{\imath}+3\hat{\jmath}+\hat{k})$
30	Find the particular solution of the differential equation $(\tan^{-1}y-x)dy=(1+y^2)dx$, given that $x=0, y=0$. OR Form the differential equation of the family of circles touching the x-axis at origin.
31	Bag I contains 1 white , 2 Black and 3 red ball , Bag II contain 2 white , 1 black and 1 red balls; Bag III contains 4 white , 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III. OR Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively.If both try to solve the problem independently, find the probability that (a) the problem is solved (b) exactly one of them Solve the problem.

32	A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?
	SECTION D
33	Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $8/27$of the volume of the sphere.
34	By using properties of determinants prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$. OR Find and use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x-y+2z=1\,;2y-3z=1\,;3x-2y+4z=2$
35	Using integration find the area of the region $\{(x,y): y^2 \le 4x, 4x^2+4y^2 \le 9\}$.
36	Find the equation of the plane through the line of intersection of the planes x +y + z =1 and 2x + 3y +4z = 5 which is perpendicular to the plane x – y +z = 0.Also find the distance of the plane obtained above ,from the origin . OR

	Find the distance of the point (1, -2, 3) from the plane x-y+z=5 measured parallel to the line $\frac{x+1}{2}=\frac{y+3}{3}=\frac{z+1}{-6}$.

SAMPLE QUESTION PAPER -6

CLASS – XII SUBJECT: - MATHEMATICS

Time allowed : - **3** hours Maximum Marks : **80**

SECTION A

(Questions 1 to 20 carry 1 mark each.)

1. A matrix has 5 elements, write all possible order it can have.
2. If $y=\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$, find $\frac{dy}{dx}$
3. Evaluate: -

$$\int_0^1 \frac{dx}{1+x^2}$$

4. Find the rate of change of area of a circle with respect to its radius r when r= 4 cm
5. Show that all the diagonal elements of a skew symmetric matrix are zero.
6. The optimum value of LPP Z=3x+4y subject to $x+y\le 3, y\le 6, x, y\le 0.$
7. Let E and F be the events with P(E) = 3/5 and P(F)= 3/10, and $P(E\cap F)=\frac{1}{5}$ then whether E and F are independent.
8. Write the smallest reflexive relation on set A= {a,b,c}
9. If $AB=6I$ then what will be A^{-1} in terms of B
10. If y= logx then $\frac{d^2y}{dx^2}$ (a) $\frac{1}{x^2}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $\frac{1}{x}$
11. Write The equation of a plane which passes through (4,-1,3)

12. Evaluate $\int \frac{x^2-1}{x^2+1}dx$

13. Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0.$

14. Find the area of the region bounded by the curve y = [x] , x axis between x =1 and x = 2

15. If the matrix $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric find the value of x.

16. Evaluate $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right)dx$

17. Find the angle between $\vec{r}.(i-2j-2k)=1$ and $\vec{r}.(3i-6j+2k)=1$

18. Find the mean of the distribution

x	0	1	3
P(X)	1/8	3/8	3/8

19. The function is differentiable or not y=|x-2| state true or false.

20. If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ find the value of x+y+xy.

SECTION B

(Questions 21 to 26 carry 2 marks each)

21. Show that the function f(x) = $4x^3 - 18x^2+27x -7$ is increasing function.

22. Let A={1,2,3,…9} and R is a equivalence relation defined on AXA as (a,b)R(c,d) $\Leftrightarrow$a+d =b+c . find the equivalence class of (2,5)

23. A can solve 90% problem and B can solve 70% problem of a book. What is the probability that a randomly chosen problem from the book will be solved by atleast one.

24. Find the foot of perpendicular on the plane 2x+y+2z+5 =0 from (2,1,0).

25. Prove that angle in semi circle is a right angle.

26. Find the vector equation of the line passing through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

SECTION C

(Questions 27 to 32 carry 4 marks each)

27. Solve for x : $- \text{Tan}^{-1}3x + \text{Tan}^{-1}2x = \pi/4$.

28. For what value of k , the following function is continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{1-cos4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

OR

Find the relationship between $'a'$ and $'b'$ so that the function $'f'$ defined by:

$$f(x) = \begin{cases} ax + 1, & if x \leq 3 \\ bx + 3, & if\ x > 3 \end{cases}$$ is continuous at $x = 3$.

29. Evaluate $\int \frac{2x}{(x^2+1)(x^2+3)} dx$

30. Solve the differential equation: $\frac{dy}{dx} + \frac{2x}{x^2+1} y = \frac{1}{\left(x^2+1\right)^2}$; $y(0) = 0$

OR

Solve the following differential equation: $\left(x^3 + y^3\right) dy - x^2 y\, dx = 0$

31. A manufacturer produces of two types of steel trunks. He has two machines A and B. For completing, the first types of trunk requires 3 hours on machine A and 3 hours on machine B, whereas the second type of trunk requires 3hours on machine A and 2 hours on machine B. Machine A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and the second type

respectively. How many trunks of each type must he make each day to make maximum profit?

32. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively.If both try to solve the problem independently, find the probability that (a) the problem is solved (b) exactly one of them . Solve the problem.

SECTION D

(Questions 33 to 36 carry 6 marks each)

33. A school wants to awards its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. 6000. Three times the award money for Hard work added to that given for Honesty amounts to Rs. 11000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method.

Or

By using properties of determinants prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

34. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R *is* $\frac{4R}{3}$. *Also* show that the maximum volume of cone is$\frac{8}{27}$ of the volume of the sphere.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area C^2 square units. Show that the maximum volume of the box is $\frac{C^3}{6\sqrt{3}}$ cubic units.

35. Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a>0$

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$

36. Find the coordinates of the foot of perpendicular and the perpendicular distance of the point P(3,2,1) from the plane $2x - y + z + 1 = 0$. Find also find the image of the point in the plane.

SAMPLE QUESTION PAPER - 7

Subject : Mathematics Class : XII

Time: 03 hrs **Full Mark: 80**

SECTION : A

(Question numbers 1 to 20 carry 1 mark each .)

(1) If the mapping f and g are given by f={ (1,2), (3,5), (4,1) } and g = { (2,3), (5,1) (1,3) }, then write fog.

(2) Write the value $\sin(\tan^{-1} a + \cot^{-1} a)$

(3) Evaluate : $\begin{vmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{vmatrix}$

(4) Find the Principle Value of $\cot^{-1}(-\sqrt{3})$.

(5) Let f: R→ R be a functions defined by f(x) = $(3 - x^3)^{\frac{1}{3}}$. Find fof(x) .

(6) A is a square matrix A of order 3 and $|4A|=K|A|$ then find the value of K.

(7) Evaluate: $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$.

(8) If A is an invertible matrix of order 3 and $|A| = 5$, then find $|adj\ A|$.

(9) Write the differential coefficient a^x.

(10) A stone is dropped into a quiet lake and waves move in a circles at speed of 5 cm/s. At the instant when the radius of the circle is 8 cm, how fast is the enclosed area increasing?

(11) Evaluate: $\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

(12) Evaluate: $\int \operatorname{cosec} x\,(\operatorname{cosec} x + \cot x)dx$

(13) $\frac{d^4y}{dx^4} = c + \left(\frac{dy}{dx}\right)^3$ $(Ans: order: 4 \;\; Deg: 1)$

(14) Find the length of the x-intercept cut off by the plane 2x + y – z = 5

(15) Find the angle between the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath}$-$2\hat{\jmath} + \hat{k}$.

(16)Find the projection of $\vec{a}$ on $\vec{b}$, if$\vec{a}.\ \vec{b}$ =8 and $\vec{b}$ =$2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$.

(17)Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z+5}{6}$.

(18) The corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0).Let Z=Px + qy where p,q > 0. Write Condition on p and q so that the min Z occurs at (3,0) and (1,1).

(19) f P(A) = 0.3,P(B) = 0.6, P(B/A) = 0.5,find P(A∪ B) .

(20) A four digits number is formed using the digits 1,2,3,5 with no repetitions. Write the probability that the number is divisible by 5.

SECTION : B

(Question numbers 21 to 26 carry 2 marks each .)

(21) Evaluate $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$,

(22) Let f : R→R such that f(x) = $\sin^2 x$ and g :R—>R such that g(x)=x find f o g (Ans: $\sin^2 x$)

(23) Solve for x $\begin{bmatrix}1 & x\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 2\end{bmatrix}\begin{bmatrix}1\\ 3\end{bmatrix} = [0]$

(24) If the function $f(x)=\begin{cases}\frac{1-\cos x}{8x^2}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at x=0 then find the value of k.

(25) A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Determine (i) k (ii) P(X<3)

(26) Find the integrating factor of the differential equation

$\frac{dy}{dx}+y\ \cot x = 2x + x^2 \cot x$ (x ≠ 0)

SECTION : C

(Question numbers 27 to 32 carry 4 marks each .)

(27) Using properties of determinant , prove that

$$\begin{vmatrix}(y+z)^2 & xy & zx\\ xy & (x+z)^2 & yz\\ xz & yz & (x+y)^2\end{vmatrix} = 2xyz(x+y+z)^3$$

(28) If $y=\sqrt{x+1}-\sqrt{x-1}$, prove that $(x^2-1)\frac{d^2y}{dx^2}+x\ \frac{dy}{dx}-\frac{1}{4}\ y=0$.

(29) Find the particular solution : $\frac{dy}{dx}-\frac{y}{x}+cosec\left(\frac{y}{x}\right)=0$; $y=0\ when\ x=1$

(30)P and Q are two points with position vectors $3\vec{a}-2\vec{b}$ and $\vec{a}+\vec{b}$ respectively. Write the position vector of a point R Which divides the line segment PQ in the ratio 2 :1 externally.

(31) A man is known to speak truth 3 out of 4 times . He throws a die and reports that it is a six . Find the probability that it is actually a six .

(32) A dealer wishes to buy a number of fans and sewing machines. He has only Rs 5760 and has space for at the most 20 items. A fan costs

him Rs 360 and a sewing machine costs Rs 240. He expects to sell a fan at a profit of Rs22 and a sewing machine for a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximise his profit? Solve it graphically.

SECTION : D

(Question numbers 33 to 36 carry 6 marks each .)

(33) A square piece of tin of side 18 cm is to be made into a box without top, bycutting a square from each corner and folding up the flaps to form the box. Whatshould be the side of the square to be cut off so that the volume of the box is the maximum possible.

(34) Integrate: $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$ or Evaluate

$\int_1^4 f(x)\,dx$, where $f(x) = |x-1| + |x-2| + |x-3|$

(35) Using integration find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

(36) Find the distance of the point (-2,3,-4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x + 12 y – 3z + 1 =0

Model Question Paper-8:

CLASS XII Subject: Maths

Max Marks: 80 Time:03 hours

	SECTION- A (01 MARK EACH)
Q.1	Find *gof* (-3) if $f(x) = \|x\|$ and $g(x) = \|5x - 2\|$
Q.2	Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$
Q.3	Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$
Q.4	Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Q.5	For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?
Q.6	If A is a Square matrix of order 3 and $\lvert A\rvert = 3$ *then find* $\lvert A(adjA)\rvert$
Q.7	If A is a sq matrix of order 3 such that $\lvert adjA\rvert = 64$. Find $\lvert A^T\rvert$.
Q.8	Find x if $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$
Q.9	Differentiate $e^{\log x}$ w. r. t. *x.*
Q.10	The radius of circle is increasing at the rate of 0.7cm/sec. What is the rate of increase of its circumference?
Q.11	Find the value of the integral**:** $\int_{-1}^{1} x^{17}\cos^4 x\, dx.$
Q.12	**Evaluate** $\int \frac{4^x}{9^x} dx$
Q.13	Form the differential equation for the curve, $y = A\cos x + B\sin x$ where *A and B* are arbitrary constants
Q.14	If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 1$and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$.Then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c}. + \vec{c}.\vec{a}$
Q.15	Find the distance between the planes 2x-y+2z=5 and 5x-2.5y+5z=20
Q.16	Find a unit vector parallel to the sum of vectors $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.
Q.17	Find the condition for λ, if the vectors$\lambda i + \hat{j} + 2\,\hat{k}, \hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar.
Q.18	If the value of the objective function z can be increased or decreased indefinitely, such solution is called _____ A. Bounded solution B. Unbounded solution C. Solution D. None of the above

Q.19	If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive event.
Q.20	A couple has 2children. Find the probability that both are boys, if it is known that one of them is a boy.
	SECTION- B (02 MARKS EACH)
Q.21	Express the matrix A = $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$, as the sum of a symmetric matrix and skew symmetric matrix.
Q.22	If $y = e^{a\cos^{-1}x}$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$.
Q.23	Find the approximate value of f(5.001), where f(x) = $x^3 - 7x^2 + 15$
Q.24	Evaluate $\int e^x \left(\frac{2+sin2x}{1+cos2x}\right) dx$
Q.25	The probabilities of A and B solving a problem are $\frac{1}{3}$ and $\frac{2}{7}$ .If all the three try to solve the problem simultaneously, Find the probability that problem is s not solved.
Q.26	Evaluate, $\int \frac{\cos 2x}{(cosx+sinx)^2} dx$
	SECTION- C(04 MARKS EACH)
Q.27	Prove that the curves x = y^2 and xy = k intersect at right angles if $8k^2 = 1$ OR If $y = x^{\sin x} + (\tan x)^x$, find $\frac{dy}{dx}$
Q.28	By using properties determinants show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ = $(1+a^2+b^2)^3$ OR

	If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB and use Hence to solve the following equations: x-y+z =4, x-2y-2z = 9, 2x+y+3z =1
Q.29	Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$
Q.30	Three vector $\vec{a}, \vec{b}$ *and* $\vec{c}$ satisfies the condition $\vec{a}+\vec{b}+\vec{c}$ $=\vec{0}$. Evaluate the quantity, $\mu = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $\lvert\vec{a}\rvert$=1, $\lvert\vec{b}\rvert = 4$ *and* $\lvert\vec{c}\rvert$=2 OR If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then show that $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar.
Q.31	A box manufacturer makes large and small boxes from a large piece of cardboard. The area of cardboard required for making one large box and one small box is $4\,m^2$ and $3\,m^2$ respectively. The manufacturer is required to make at least 3 large boxes and at most twice as many small boxes as large boxes. If $60\,m^2$ of cardboard is in stock, and if profit on the large and small boxes are ₹3 and ₹2 respectively. How many of each should be made in order to maximize the total profit? Formulate the above LPP and then solve it graphically
Q.32	There are two bags withfirst bag containing 6 green and 3 red balls. The second bag contains 5 green and 4 red balls. One ball is transferred from first bag to second bag. Thereafter, one ball is drawn from first bag and it is found to be red. What is the probability that transferred ball was red?

	SECTION- D(06 MARKS EACH)
Q.33	Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ OR $Integrate \int_1^2 (x^2 + x + 2) dx \; by \lim it \; as \; a \; sum.$
Q.34	Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration
Q.35	Find the image of the point P(3, 2, 1) in the plane $2x - y + z + 1 = 0$. **OR** Find the equation of the plane through the line of intersection of the planes $\vec{r}.(2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which are at unit distance from origin.
Q.36	Find a particular solution of the differential equation, $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}, \quad where \; y(1) = 0$

Suggested Activities and Projects to be conducted

(ANY TEN) by Kolkata Region

Class XII (2019-20)

Relations & Functions (already given with the topic)

Continuity and Differentiability

1. To find analytically the limit of a function f (x) at x = c and also to check the continuity of the function at that point.
2. To verify Rolle's Theorem and Lagrange's Mean Value Theorem.
3. To understand the concepts of decreasing and increasing functions.
4. To understand the concepts of local maxima, local minima absolute maximum and minimum values of a function and point of inflection.

5. Different application based problem on Maxima And Minima.

Integrals:

1. To evaluate the definite integral as the limit of a sum and verify it by actual integration.

Vectors

1. To verify geometrica $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

2. To verify that angle in a semicircle is a right angle, using vector method.

Three - dimensional Geometry

1. To demonstrate the equation of a plane in normal form.

2. To verify that the angle between two planes is the same as the angle between their normals.

3. To measure the shortest distance between two skew lines and verify it analytically.

List of suggested Projects

1. Study the nature of Mathematics and make a project where three aspect of nature of Mathematics – formalism, logic, intuition is applied for the development of mathematics.

2. History of foreign Mathematicians such as Cantor, Pythagoras, Thales, Euclid, Appollonius, Descartes, Fermat, Leibnitz, Euler, Fibonac, Gauss, Newton.

3. Mathematics and Chemistry: Study structure of organic compounds.

4. To minimise the cost of the food, meeting the dietary requirements of the staple food of the adolescent students of your school.

5. To explain the computation of conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice.

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